CM322 COMPLEX ANALYSIS CLASS TEST 1, 2010

You have 45 minutes for this test. You may not use calculators or formula sheets. All questions will be marked. You may use your own paper but must put the final answers on this sheet.

PRINT YOUR NAME HERE:

PRINT YOUR STUDENT NUMBER HERE:

1. In each of the following cases determine whether the set Ω is connected, open and/or bounded:
(a) $\Omega = \{z : \operatorname{Re} z > 0, \operatorname{Im} z > \sin(\frac{1}{\operatorname{Re} z})\}$, **(b)** $\Omega = \{z : 0 < |z| < 1\}$.

2. State precisely what it means to say that a function $f(z)$ is
(a) differentiable, **(b)** entire.
If $f(z)$ is entire and $f'(z) = 0$ at any point $z \in \mathbb{C}$, prove that f is identically equal to a constant. (Any results from real analysis may be used without justification.)

3. Prove rigorously that the function $f(z) = \operatorname{Re} z$ is nowhere differentiable.
(In this question you **may not** use the Cauchy–Riemann equations.)

4. Define the radius of convergence $\hat{R}$ of a power series $\sum_{n=0}^{\infty} a_n z^n$. Can $\hat{R}$ be equal to 0 or ∞? Is there a power series which converges for all $z \in \mathcal{D}(0,2)$ and diverges for all $z \in \mathbb{C}$ with $|z| = 2$? Justify your answers by giving examples or/and quoting appropriate results from the course.

5. Define the exponential function and prove that $\exp(z+w) = \exp z \, \exp w$.
(You may use any results on series and differentiation rules without justification.)

6. Let $T(z) = \frac{az+b}{cz+d}$ be a Möbius transformation with $c \neq 0$. Prove that T can be represented as the composition of four basic transformations: translation, dilatation, rotation and inversion.