

1. Let A be a non-empty open subset of \mathbb{C} , and let A^* denote the set given by

$$A^* = \{w \in \mathbb{C} : \bar{w} \in A\}.$$

Thus, A^* is obtained from A by reflection in the real axis. Show that the set A^* is also open.

Assume that a function f is analytic in A . Prove that the function $g(z) = \overline{f(\bar{z})}$ is analytic in A^* . If $f(z) = c_0 + c_1z + \cdots + c_nz^n$, what is $g(z)$? What if $f(z) = e^{iz}$?

2. Which of the following functions is differentiable (in the complex sense)? Explain.

$$(i) f_1(z) = |z|^2; \quad (ii) f_2(z) = \frac{z}{1 + |z|}; \quad (iii) f_3(z) = \frac{z + 1}{z - 1}.$$

3. Does there exist a function $f(z)$ defined on \mathbb{C} and satisfying the Cauchy–Riemann equations everywhere, such that $\operatorname{Re} f(x + iy) = 3x^2y - y^3$?
4. Let f be a complex function holomorphic on some open disc D . Assume that there exist real constants a , b and c such that $a \operatorname{Re} f(z) + b \operatorname{Im} f(z) = c$ for all z . Show that $f(z)$ is constant on D .