**1.** A set $\Omega \subset \mathbb{C}$ is said to be (*convex* if the line segment joining every two points $z_1, z_2 \in \Omega$ lies in $\Omega$. Show that any convex set is path-connected.

**2.** Sketch the following subsets of $\mathbb{C}$ and decide whether they are (i) open; (ii) closed; (iii) path connected; (iv) bounded; (v) convex.

$$A = \{\, z \,:\, 1 < |z-1| < 2 \,\}; \qquad B = \{\, z = x + iy \,:\, 0 < |x+y| \leq 2 \,\};$$

$$C = \{\, z \,:\, |\arg(z-1)| \leq \frac{\pi}{4} \,\}; \qquad D = \{\, z \,:\, 0 < \arg z < \pi/2 \,,\, |z| = 1 \,\}.$$

**3.** Show that if $a$ and $z$ both belong to the unit disc $D(0,1) = \{\, z : |z| < 1 \,\}$, then so does the complex number

$$w = \frac{z-a}{1-\overline{a}\, z}\,.$$

[Hint: calculate $1 - w\overline{w}$.]

**4.** Find the radius of convergence of the following power series:
(i) $\sum_0^\infty nz^n$, (ii) $\sum_0^\infty \frac{z^n}{n^2}$; (iii) $\sum_0^\infty \frac{n^2}{2^n} z^n$; (iv) $\sum_0^\infty (n!) z^n$.
You may use the identity $\lim\limits_{n\to\infty} \frac{\ln n}{n} = 0$ without justification.