1. Let $a, b, c, d \in \mathbb{R}$ and $ad - bc > 0$. Show that the Möbius transformation

$$z \mapsto T(z) = \frac{az + b}{cz + d}$$

preserves the extended real line $\widehat{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ and maps the upper half plane $\mathcal{H} = \{\, x + iy \mid y > 0 \,\}$ bijectively to itself. What is the image of the lower half plane $\mathcal{L} = \{\, x + iy \mid y < 0 \,\}$? Find also the image point $T(\infty)$.

2. Find the image of the line $\{\, z : \operatorname{Re} z = 1 \,\}$ under the inversion map

$$z \mapsto U(z) = 1/z.$$

3. Find a Möbius transformation which maps the disc $\mathcal{D}(i, 1)$ onto the left half-plane $\{\, w : \operatorname{Re} w < 0 \,\}$ and such that the point $z = 0$ is mapped to the point $w = 0$.

Hint: work it out as a composition of specific mapping, including $z \mapsto z - i$ and $z \mapsto \frac{iz+i}{1-z}$.

4. Show that the infinite strip $G = \{\, z \in \mathbb{C} : 1 < \operatorname{Im} z < 4 \,\}$ is an open set in $\mathbb{C}$.

5. For each integer $n \in \mathbb{N}$, let $G_n = \mathcal{D}(0, \frac{1}{n})$. Find the intersection $\bigcap_{n \in \mathbb{N}} G_n$. Deduce that the intersection of an infinite number of open sets need not be open.

Give an explicit example of a collection of open sets in $\mathbb{C}$ whose intersection is neither open nor closed.