1. We have $\dfrac{\tan x}{1+\tan x} = \dfrac{\sin x}{\sin x + \cos x}$. If $x < \pi/2$ and $x \to \pi/2$ then $\sin x \to 1$ and $\cos x \to 0$. Applying the theorem about the limits of sums and quotients, we obtain $\displaystyle\lim_{x\to\pi/2-0} \frac{\tan x}{1+\tan x} = \frac{1}{1+0} = 1.$

2. The composition and sum of continuous functions are continuous functions. The quotient is also a continuous function provided that the denominator does not vanish. Since sin and cos are continuous functions, $\tan = \frac{\sin}{\cos}$, all polynomials are continuous and $1+x+x^2 \neq 0$, we see that $\dfrac{\tan(x+x^2)}{1+x+x^2}$ is continuous everywhere with the exception of points x satisfying $\cos(x+x^2) = 0$. In the interval $[0,1]$ there is only one such point, namely, the solution to the equation $x + x^2 = \pi/2$.

3. The first part is proved by evaluating the left hand side at the end points and applying the intermediate value theorem. The precise formula for the solution is obtained by substituting $s = \cos x$.

4. Since $(1+x^2)^{-1} \to 0$ as $|x| \to \infty$, for large values of $|x|$ the function $\sin x - (1+x^2)^{-1}$ is positive near the points $\pi/2 + 2n\pi$ and is negative near the points $-\pi/2 + 2n\pi$. By the intermediate value theorem, there is a solution in each interval $[-\pi/2 + 2n\pi, \pi/2 + 2n\pi]$, where $n = 1, 2, \ldots$

5. continuous and unbounded on $[0,1)$;
   YES e.g. $f(x) = 1/(1-x)$.

6. continuous and unbounded on $[0,1]$;
   NO, the range of a continuous function on a closed bounded interval is a closed bounded interval.

7. continuous on $[0,1]$ and $\{f(x) : x \in [0,1]\} = (0,1)$;
   NO, for the same reason

8. continuous on $(0,1)$ and $\{f(x) : x \in (0,1)\} = [0,1]$;
   YES, e.g. $f(x) = \sin(4\pi x)$.

9. continuous on $(0,1)$ and $\{f(x) : x \in (0,1)\} = [0,1] \cup [3,4]$.
   NO, by the intermediate value theorem.