1. By the ratio test, the series $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ converges. Since $0 < \frac{2^n-n}{3^n+n^2} \le \left(\frac{2}{3}\right)^n$, the comparison theorem implies that the series $\sum_{n=1}^{\infty}\frac{2^n-n}{3^n+n^2}$ is also convergent.

2. Since $\frac{n^n}{n!}$ does not converge to zero as $n \to \infty$, the series diverges.

3. $\left|\frac{(-1)^n\,n^k}{1+2^n}\right| \le \frac{n^k}{2^n} = c_n\,(\sqrt{2})^{-n}$. We know that $c_n \to 0$ as $n \to \infty$ (see the previous exercise sheet). Therefore, by ratio test, the series $\sum_{n=1}^{\infty} c_n\,(\sqrt{2})^{-n}$ converges. Now the comparison theorem implies that the series $\sum_{n=1}^{\infty}\frac{(-1)^n\,n^k}{1+2^n}$ is also convergent.

4. We have $\lim\frac{n-1}{n+1} = 1$. The ratio test implies that the series converges for all x such that $|x+1| < 2$. If $|x+1| \ge 2$ then the sequence $\frac{(n-1)\,(x+1)^n}{(n+1)\,2^n}$ does not converge to 0 and, consequently, the series diverges.