1. Let $b > 0$. If $a > 0$ then, by (**A13**), $ab > a \times 0$ where the right hand side is equal to 0 (see course notes).

2. By (**A6**) and (**A8**), $s = at$ with $a = s\,t^{-1} = t^{-1}s$. The axiom (**A13**) implies that $t^{-1}s > 1$ if and only if $s > t$.

3. By (**A6**) and (**A9**),

$$(a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$$

$$= a^n + a^{n-1}b + a^{n-2}b^2 + \ldots + ab^{n-1} - a^{n-1}b - a^{n-2}b^2 - \ldots - ab^{n-1} - b^n = a^n - b^n\,.$$

   A more "scientific" (and more elegant) proof is by induction in n.

4. Assume that $x^3 = a$ and $y^3 = a$. Then, by the above,

$$(x-y)\,(x^2 + xy + y^2) = 0\,.$$

   The sum in the brackets is strictly positive because every term is positive (see Question 1). Using (**A8**) and (**A13**), we can multiply both parts by $(x^2 + xy + y^2)^{-1}$. Then we obtain

$$x - y = 0.(x^2 + xy + y^2)^{-1} = 0.$$

   Now (**A13**) implies that $x = y$.

5. If $|x| \geq |y|$ then $|\,|x| - |y|\,| = |x| - |y|$. Since $x = y - (y - x)$, the triangle inequality implies that $|x| - |y| \leq |y - x| = |x - y|$. If $|x| \leq |y|$ then $|\,|x| - |y|\,| = |y| - |x|$. Since $y = x - (y - x)$, the triangle inequality implies that $|y| - |x| \leq |x - y|$. The proof uses the axioms (**A2**), (**A12**) and the definition of the modules.

6. By the Archimedean Property (see course notes), we have $\delta^{-1} < n$ for some $n \in \mathbf{N}$. Multiplying both parts of this inequality by $n^{-1}\delta$, we obtain $n^{-1} < \delta$. Here we have used the axioms (**A6**), (**A8**) and (**A13**).