

1. Let  $a$  and  $b$  be real numbers such that  $b > a$ , and let  $f$  and  $g$  be continuous functions on the interval  $[a, b]$ . State whether the following statements are correct. In each case write down few words of explanation or give a counterexample.
  - 1a. If  $f(x) > g(x)$  for all  $x$  then  $\int_a^b f(x) dx > \int_a^b g(x) dx$ .
  - 1b. If  $|f(x)| > 2|g(x)|$  for all  $x$  then  $\int_a^b f(x) dx \geq 2 \int_a^b g(x) dx$ .
  - 1c. If  $c \in (a, b)$  then  $\int_a^b |f(x)| dx \geq |\int_a^c f(x) dx|$ .
2. Give a complete and careful proof that  $\lim_{n \rightarrow \infty} \int_1^n e^{-x^2} dx$  exists and is finite. You may use any theorems about sequences without proof, but you must not use the integral comparison test.
3. Using the fundamental theorem of calculus and the chain rule, evaluate the second derivative

$$\frac{d^2}{dx^2} \left( \int_0^{x^2} \frac{1}{1+t^3} dt \right).$$