1. Suppose that f is differentiable on the interval (a, b). Prove that if $f'(x) > 0$ for all $x \in (a, b)$ then f is strictly monotonically increasing in the sense that $f(u) > f(v)$ whenever $u > v$. Write down an example that proves the converse statement is false: there exists a differentiable function f such that $f(u) > f(v)$ whenever $u > v$ but $f'(x) > 0$ for all x is false.

2. By applying the mean value theorem to $f(x)\,\mathrm{e}^{-cx}$, prove that if f is differentiable on $\mathbf{R}$ and $f'(x) = cf(x)$ for all $x \in \mathbf{R}$ then $f(x) = \mathrm{e}^{cx} f(0)$ for all $x \in \mathbf{R}$.

3. Use an appropriate theorem to find the range of values of $x \in \mathbf{R}$ for which each of the following power series converges.

(3a) $$\sum_{n=1}^{\infty} \frac{n^9 - 1}{n^8 + 1}\, x^n$$

(3b) $$\sum_{n=0}^{\infty} \frac{2^n + 3^n}{4^n + 5^n}\, x^n$$

(3c) $$\sum_{n=0}^{\infty} \frac{(2n)!\, x^n}{(n!)^2}$$