

1. Find the maximum and minimum values of the function

$$f(x) = 2x^5 - x^3$$

on the interval $[0, 1]$, and sketch the graph of the function on this interval.

2. Prove that the derivative of the function

$$f(x) = (\sin x)^2 + x^2$$

only vanishes at one point in $[-\pi, \pi]$. Find the maximum and minimum values of the function $f(x)$ on the interval and sketch its graph.

3. Using the identities $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$, find the derivative of $\tan x$. Use the chain rule and the identity

$$\arctan(\tan x) = x$$

to show that the derivative of $f(x) = \arctan x$ is equal to $(1 + x^2)^{-1}$.

4. Use the mean value theorem to obtain the best upper and lower bounds on $\arctan(1.1)$ that you can. Note that $\arctan(1) = \pi/4$.