

Calculate the derivatives of each of the following two functions. Find all of the local and global maximum and minimum values, if there are any, of the functions on the intervals given. Sketch the graphs of the functions in these regions **and explain your results!**

1. $f(x) = \frac{1}{1+\log(x)^2}$ on $(0, e]$;

2. $f(x) = xe^{-x^2/2}$ on $[0, \infty)$;

3. Evaluate the following derivative (you must state the conditions under which the result holds)

$$\frac{d}{dx} \left(\frac{f(x)^2}{g(x)^3} \right).$$

4. Evaluate the derivatives of the functions $f(x) = x^x$ for all real numbers $x > 0$. You will need to use the standard rules for differentiating the exponential and log functions.

5. Calculate the derivative from the left and derivative from the right at $x = 0$ of the function

$$f(x) = \tan(2|x| + x).$$

6. The function $f(x)$ is defined for all $x \in \mathbf{R}$ by

$$f(x) = \begin{cases} x^2(\sin(1/x)) & \text{if } x \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the derivative of $f(x)$ at all non-zero x . Using only the ε, δ definition of differentiation prove that f is differentiable at $x = 0$ and find its derivative at that point. Sketch the graph of the derivative. Is the derivative continuous at $x = 0$?

7. Let $g(x) = e^{-1/x^2}$ if $x \neq 0$ and $g(0) = 0$. Calculate the first two derivatives of $g(x)$ at $x = 0$. **Explain your answer!**