1. Evaluate
$$\lim_{x\to\pi/2-0} \frac{\tan x}{1+\tan x}$$
and explain why you answer is correct.

2. Prove that the function
$$f(x) = \frac{\tan(x+x^2)}{1+x+x^2}$$
is continuous at every $x \in [0,1]$ except for one point, which you should identify.

3. Use the intermediate value theorem to prove that the equation
$$(\sin x)(\tan x) = 1$$
has a solution in the range $\pi/4 < x < \pi/3$. Prove that the exact solution is
$$x = \arccos\left(\frac{\sqrt{5}-1}{2}\right).$$

4. Sketch the graphs of the functions on the two sides of the equation below. Use the intermediate value theorem to prove that the equation
$$\sin x = \frac{1}{1+x^2}$$
has an infinite number of real solutions.

In each case below, decide whether it is possible for a function f to be

5. continuous and unbounded on $[0,1)$;

6. continuous and unbounded on $[0,1]$;

7. continuous on $[0,1]$ with the range $\{f(x) : x \in [0,1]\} = (0,1)$;

8. continuous on $(0,1)$ with the range $\{f(x) : x \in (0,1)\} = [0,1]$;

9. continuous on $(0,1)$ with the range $\{f(x) : x \in (0,1)\} = [0,1] \cup [3,4]$.

In each case justify your answer by giving an example or by quoting a theorem that shows that no example can exist.