1. Prove that, for every $\delta > 0$, there exists $n \in \mathbf{N}$ such that $n^{-1} < \delta$.

2. Let $a_1 = 1$ and $a_{n+1} = \dfrac{a_n}{a_n + 2}$ for all $n \geq 1$. Prove that the sequence is decreasing and find its limit.

3. Making references to any theorems about convergence which you use, evaluate $\lim_{n\to\infty} \dfrac{2n^3 + n^2 + n - n^5}{n^3 + 2n - 1 + 2n^5}$ and $\lim_{n\to\infty} \dfrac{2^n + 3}{3^n + 2}$.

4. Let $k \in \mathbf{R}$ and $b > 1$. Prove that $\lim_{n\to\infty} n^k b^{-n} = 0$.

5. Let $\{a_n\}$ be a convergent sequence and $\lim_{n\to\infty} a_n = a$. Prove that every subsequence of $\{a_n\}$ converges to a.

6. Show that c is an accumulation point if and only if any open interval of the form $(c - \varepsilon, c + \varepsilon)$ contains infinitely many elements of the sequence $\{a_n\}$.