CM221A ANALYSIS CLASS TEST 1, 2010

You have 45 minutes for this test. You may not use calculators or formula sheets. You should leave a question blank if you do not know the answer. You may use your own paper but must put the final answers on this sheet.

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1. Prove that every positive real number x, there exists a positive integer n such that $\frac{1}{n} < x^2$. You may NOT use the Archimedean property without proof.

2. Let $a \in (-1, 1)$. Using the principle of induction, show that
$a + a^3 + a^5 + \ldots a^{2n-1} = \dfrac{a - a^{2n+1}}{1 - a^2}$ for all positive integers n.

3. What does it mean to say that a sequence $\{a_n\}$ converges? Write down a precise definition, using $\varepsilon > 0$. Give an example of a sequence which does not converge.

4. What does it mean to say that a series $\sum_{n=1}^{\infty} a_n$ is
(i) convergent,
(ii) absolutely convergent?
Explain the relation between these two notions. Give an example of a series which converges but is not absolutely convergent.

5. In each of the following cases, determine the range of x for which the series is convergent. Consider all possible values of x and explain what tests and/or theorems you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$ **(c)** $\displaystyle\sum_{n=1}^{\infty} (\sin x)^n$.