

You have 45 minutes for this test. You may not use calculators or formula sheets. You should leave a question blank if you do not know the answer. You may use your own paper but must put the final answers on this sheet.

PRINT YOUR NAME HERE:

PRINT YOUR STUDENT NUMBER HERE:

PRINT YOUR TUTOR'S NAME OR GROUP NUMBER HERE:

1. Let f be a differentiable function on $[a, b]$. What does it mean to say that f has a local maximum at a point $c \in (a, b)$? Prove that $f'(c) = 0$ whenever c is a local maximum.

2. Find the minimum and the maximum values of the function $f(x) = x + \cos x$ on the interval $[0, \pi]$.

3. State and prove Rolle's theorem (any results about maxima and minima of continuous and differentiable functions may be used without justification).

4. Give an example of a function which is continuous at a point c but is not differentiable at c .

5. State (but do not prove) Taylor's theorem.

6. Use Taylor's theorem with $a = 0$ to write down the first three terms of the power series of the function $f(x) = \frac{1}{2 - e^x}$.

7. Apply a suitable theorem to find out the values of x for which the power series $\sum_{n=0}^{\infty} \frac{x^n}{n! + (2n)!}$ is absolutely convergent.