



# Magnetic flux penetration mechanisms in inductive three-dimensional Josephson-junction network

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## Abstract

The time evolution of magnetic flux distribution in a  $8 \times 8 \times 8$  Josephson-junction network is numerically followed, after the network's lower threshold field is exceeded. According to results, Josephson network vortices first form at network edges as bent. With time the vortices straighten and find their stationary positions inside the network. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Granularity; Josephson junction arrays; Magnetic properties

In this paper we present a numerical simulation of the time evolution of magnetic flux distribution during the entering process in three-dimensional Josephson-junction networks (3D-JJN). The penetration dynamics have already been studied in two-dimensional JJN [1] and, lately, in type-II superconductors [2]. These studies found vortices to form at the sides of the two-dimensional (2D) network. However, real samples are three-dimensional objects. Therefore, since the 3D-JJN act as a model for intergranular magnetization in granular superconductors in the low temperature and low-field limit [3], we chose to use 3D-JJN to study the flux penetration in this class of superconductors.

Our 3D-JJN model consists of  $n_x \times n_y \times n_z$  elementary cubic cells. Each cell contains 12 resistively shunted ideal Josephson-junctions, which are characterized by a maximum Josephson current  $I_J$  and a gauge-invariant phase difference  $\varphi_\eta(\mathbf{r})$ , where  $\eta$  is the direction along which a junction is lying in the cell at the position  $\mathbf{r}$ . The magnetic flux across a face ( $\mu\nu$ ) of a cell at  $\mathbf{r}$  is  $\phi_{\mu\nu}(\mathbf{r})$ , the branch current (i.e. the current through a junction) is  $I_\mu(\mathbf{r})$ . For overdamped junctions we can write

$$I_\mu(\mathbf{r}) = \frac{\phi_0}{2\pi R} \frac{d\varphi_\mu(\mathbf{r})}{dt} + I_J \sin \varphi_\mu(\mathbf{r}),$$

which, together with flux quantization condition for zero-field cooling  $2\pi\phi_{\mu\nu}(\mathbf{r})/\phi_0 = \varphi_\nu(\mathbf{r} + a\hat{\mu}) - \varphi_\nu(\mathbf{r}) - \varphi_\mu(\mathbf{r} + a\hat{\nu}) + \varphi_\mu(\mathbf{r})$ , and the flux-current linkage with mutual inductive coupling

$$\phi_{\mu\nu}(\mathbf{r}) = \frac{\mu_0}{4\pi} \oint_{\mu\nu} \int_{r'} \frac{I_\eta(\mathbf{r}') dI_\eta(\mathbf{r}')}{d(I_\eta(\mathbf{r}), I_\eta(\mathbf{r}'))} \cdot dI_\eta(\mathbf{r}) + \phi_e,$$

form the set of non-linear ordinary differential equations, whose boundary conditions are obtained by demanding no currents outside the network. In the above  $\phi_0$  is the magnetic flux quantum,  $\phi_e$  is the external magnetic flux,  $a = 10 \mu\text{m}$ , and  $d(I_\eta(\mathbf{r}), I_\eta(\mathbf{r}'))$  is the distance between the components  $I_\eta(\mathbf{r})$  and  $I_\eta(\mathbf{r}')$ . The set is solved using the fourth-order Runge–Kutta method with adaptive time step. The 3D-JJN model is reported in detail in Refs. [3,4].

Flux penetration dynamics in a 3D-JJN was studied by increasing the field in steps  $\Delta\psi_e = 0.1\pi$ , where  $\psi_e = 2\pi\phi_e/\phi_0$ , and letting the network to find its quasistationary state during a time interval  $\Delta t = 500 L/R$  ( $1 L/R \approx 10^{-11}$  s, with  $L = 9 \times 10^{-12}$  H [4] and  $R = 1 \Omega$ ). After  $\psi_e$  exceeds the lower threshold flux of the network  $\psi_{th}$ , the entering process ensues. Let us now study the  $8 \times 8 \times 8$  network with  $\beta = 2\pi LI_J/\phi_0 = 1$  when the field is parallel to the  $z$ -axis of the network after  $\psi_e$  is increased from  $\psi_e = 0.6\pi$  to  $0.7\pi$  at time  $t = 0$ , ( $0.6\pi < \psi_{th} < 0.7\pi$ ). Fig. 1 shows flux distributions at times  $t = 0, 10, 20, 30, 40$  and  $500 L/R$ , in (a), (b), (c),

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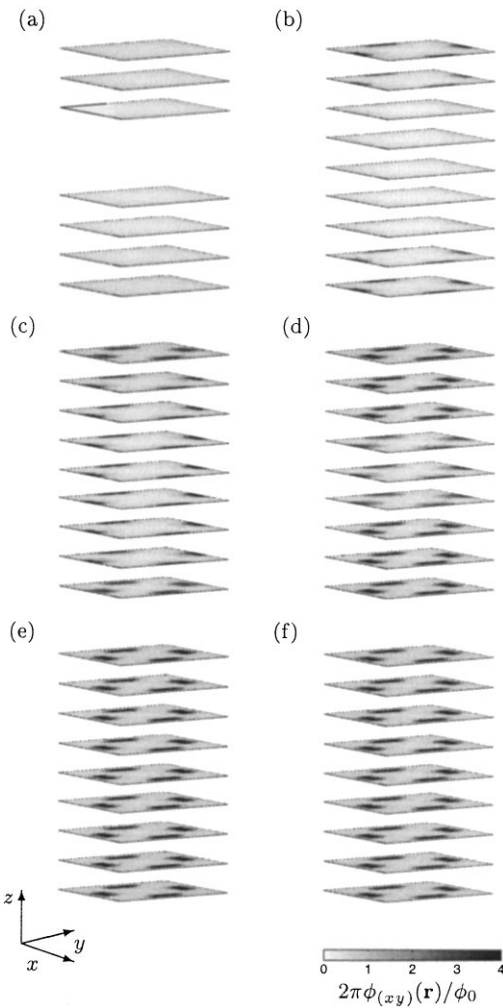


Fig. 1. Magnetic flux distribution in  $8 \times 8 \times 8$  network at times  $t = 0$  (a),  $10$  (b),  $20$  (c),  $30$  (d),  $40$  (e), and  $500$  (f)  $L/R$ , after  $\psi_c$  is increased from  $\psi_c = 0.6\pi$  to  $\psi_c = 0.7\pi$ .

(d), (e), and (f), respectively. In the case  $t = 0 L/R$ , the magnetic flux is condensed at the corners and decays exponentially from sides to center. At  $t = 10 L/R$ , the phase-slip has started in the junction at the center of the

edges on the  $xy$ -planes  $k = 0$  and  $8$ , ( $k$  denotes the position and the orientation of the layer). For  $\beta = 1$ , the vortex spreads over several loops. Therefore, the phase-slip has triggered also the neighboring junctions, and the magnetic flux starts to enter the network on planes  $k = 1$  and  $7$ . Here then a Josephson network vortex starts to form. With time, the vortex forms on the bottom and uppermost  $xy$ -planes of the network, as seen in Fig. 1(a–f). At  $t = 20 L/R$  (Fig. 1(c)), almost the whole vortex is already localized on layers  $k = 0$  and  $8$ , whereas in the center layers penetration has just started. Thus, the network vortex is bent. When  $t = 30 L/R$ , the vortex is already formed on the layers  $k = 0$  and  $8$ , Fig. 1(d). However, the entering process continues on the center planes. Finally, at  $t = 40 L/R$ , the vortex is straight and the process is almost completed, cf. Fig. 1(e) where the magnetic flux distribution is not far from that of the quasistationary state shown in Fig. 1(f) for  $t = 500 L/R$ .

In conclusion, in  $8 \times 8 \times 8$  network with  $\beta = 1$ , penetration of Josephson network vortices starts at the center of the sides at top and bottom layers and then continues through all the remaining layers. Thus, the penetrating Josephson network vortex is at first bent, acquiring a straight final configuration only after a characteristic time of the network.

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