

## Modified Arrhenius formula for the time decay of magnetic states in type II superconductors

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We study the problem of the time decay of magnetic states in type-II superconductors by starting from the general expression of the Arrhenius formula as derived from classical stochastic mechanics. By appropriately writing the potential energy for a fluxon in the presence of a pinning center, we find that the attempt frequency in the Arrhenius formula depends on the current density  $J$  in such a way that the dissipation phenomena due to creep mechanisms approach zero for vanishing values of  $J$ .

### 1. INTRODUCTION

In the study of the vortex dynamics in high- $T_c$  superconductors a non-linear increase of the creep barrier with decreasing current density has been observed by many groups [1-2]. In particular, from magnetic relaxation measurements, Maley *et al.* [1] have reported a sharp increase of the effective activation energy  $U$  as the current density  $J$  decreases. Analogous results have been found by Zeldov *et al.* [2], from transport measurements. The experimental results can be analysed within the collective pinning theory [3], which predicts the following inverse power law dependence of the potential energy from  $J$ :  $U(J) = U_i (J/J_c)^{-\mu}$ , where  $J_c$  is the critical current density,  $U_i$  is the effective activation energy for  $J = J_c$ , and the value of the exponent depends on the dimensionality of the system and the particular flux creep regime. In this model, though, the effective barrier height grows indefinitely as  $J$  goes to zero. Other models [4] predict the same diverging behavior for vanishingly small current densities.

In the present work we tackle the problem *ab initio* starting from the general expression of the Arrhenius formula for the escape probability of a single flux quantum by classical stochastic mechanics, in order to analyse in details the creep problem for independent fluxons. Therefore, by re-analysing the problem and starting from a particular shape of the potential well, which determines the current dependence of the flux decay problem, we find that the Arrhenius formula may be generalized in the following simple way:

$$\frac{1}{\tau} = \nu(J) \exp\left(-\frac{\Delta U_J}{k_B T}\right) \quad (1)$$

where  $k_B$  is the Boltzmann constant,  $\Delta U_J$  is the potential barrier height, and  $\nu = \nu(J)$  is the attempt frequency. In particular, we find that, for pinning centers larger than the coherence length  $\xi$ , the characteristic frequency  $\nu$  goes to zero for vanishingly small current density values. In this way, dissipation due to creep mechanisms becomes negligible for decreasing values of  $J$ .

### 2. MODIFIED ARRHENIUS FORMULA

We start our analysis from a potential energy  $U_J(x)$  which takes account of the presence of the current density  $J$  and of a finite size  $2l$  of the pinning center, so that we assume:

$$\frac{U_J(x)}{U_o} = \left( \tanh\left(\frac{x-l}{\xi}\right) - \tanh\left(\frac{x+l}{\xi}\right) - \epsilon_o \frac{J}{J_c} \frac{x}{\xi} \right) \quad (2)$$

where  $\xi$  is the coherence length and  $\epsilon_o$  can be supposed to be equal to 1. By adopting the same assumptions as in the usual derivation of the Arrhenius formula, following Gardiner [5], the attempt frequency can be determined by the geometric mean of the curvatures of the potential well at the local minimum  $x_{min}$  and at the local maximum  $x_{max}$  of the potential. We therefore need to first numerically find the local extrema of the

potential by setting  $U'_j(x) = 0$  where the prime stands for the first derivative with respect to  $x$ . We then set:

$$v(J) = \frac{1}{2\pi\beta} \sqrt{U''_j(x_{\min})U''_j(x_{\max})} \quad (3)$$

where  $\beta$  is the damping constant which is supposed to be  $\beta=1$  [5]. The second derivative  $U''_j(x)$  can be analytically found, so that the characteristic frequency  $v$  can be expressed completely in terms of  $J$ . In Fig.1 we report the  $v$  vs.  $J$  dependence for the value of the ratio  $l/\xi=10$ . In the inset we show the current dependence of the resulting potential barrier height  $\Delta U_j$ , where  $\Delta U_j = U_j(x_{\max}) - U_j(x_{\min})$ , for the same value of the ratio  $l/\xi$ . In this figure the horizontal line  $\Delta U_j = k_B T$  is traced. This line marks the lower limit of the range of validity of Eq.(3), since the Arrhenius formula is derived under the following assumption:  $k_B T \ll \Delta U_j$ . Therefore, care must be taken in applying these results for  $J$  close to  $J_c$ .

### 3. RESULTS

Having consistently derived the attempt frequency  $v(J)$  in the Arrhenius formula, we can find the electric field  $E_{\text{creep}}$  due to flux creep in the sample, and compare it with the corresponding quantity calculated without taking into account the current density dependence of the frequency  $v$ . This can be accomplished by simply setting:

$$E_{\text{creep}} = E_0 \frac{v(J)}{v_0} \exp\left(-\frac{\Delta U_j}{k_B T}\right) \quad (4)$$

and by substituting  $v(J)$  with a constant value in order to compare the two ways of expressing the  $E$  vs.  $J$  dependence, as specified before. In Fig.2 we, therefore, show the  $E$  vs.  $J$  dependence in the two cases, for the values of the ratio  $l/\xi = 4$  and 10.

Finally, we obtain, for  $l/\xi \gg 1$  and  $J \rightarrow 0$ , much smaller values of  $E_{\text{creep}}$  than those reported in the literature neglecting the frequency dependence on the current density.

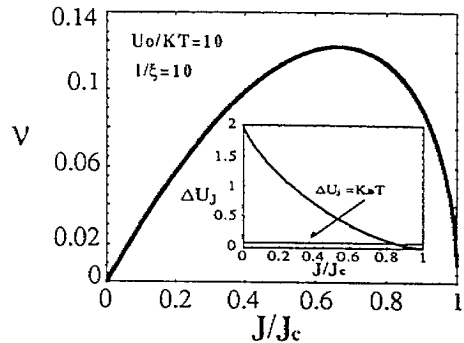


Figure 1.  $v(J)$  curve for  $l/\xi=10$ . In the inset  $\Delta U_j$  vs.  $J$  is showed for the same value of the ratio  $l/\xi$ .

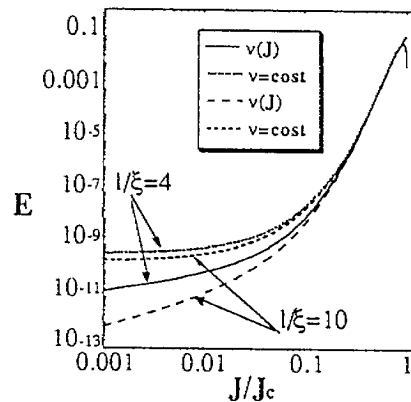


Figure 2.  $E_{\text{creep}}$  vs.  $J$  for  $l/\xi = 4, 10$  with  $v = v(J)$  and  $v = \text{const}$ .

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