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Cubic structures of Josephson coupled superconducting spheres: three-dimensional models of granular superconductors (*)

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Summary. — The magnetic response of a simple three-dimensional Josephson junction network is studied. The model network represents the circuital equivalent of a physical system consisting of eight spherical grains located at the vertices of a cube. We derive the equations for flux transitions in the system, and determine the lower threshold field.

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1. – Introduction

The transport critical current in high-T_c superconducting systems is considerably limited by the microstructural characteristics of these materials [1]. Indeed, if we schematize these systems as a collection of weakly-coupled superconducting islands, we may adopt Josephson junction network models to describe their electrodynamic response. With the aid of these models one can show that the maximum Josephson currents of the single junctions are natural upper bounds for the transport critical current [2]. Even though granularity in high-T_c superconducting systems plays against large scale and practical applications of these materials, new and interesting phenomena may arise from the study of the magnetic properties of granular systems. For example, one can show that the particular multiply connected structure of granular samples may give rise to the so-called Wohlleben effect [3, 4], which is sometimes referred to as *paramagnetic Meissner effect*. As far as the magnetic properties of these materials are concerned, one usually adopts one-dimensional (1D) [5, 6] or two-dimensional (2D) [7-9] junction arrays. Only few recent works have appeared on three-dimensional (3D) Josephson junction arrays [10, 11].

In the present work we study the magnetic response of a particular 3D Josephson junction network: twelve junctions, each one located at the midpoint of a cube side. This

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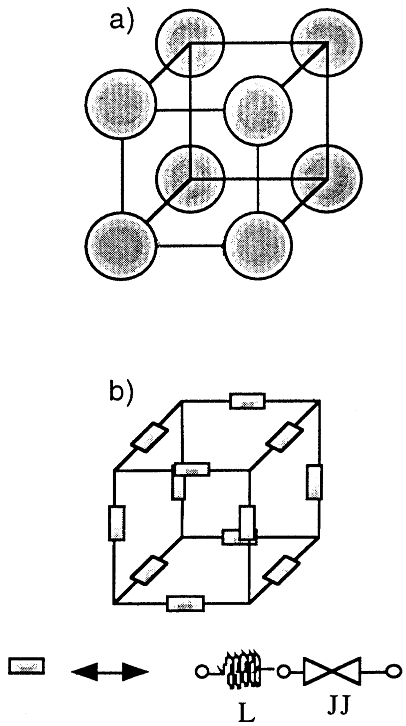

Fig. 1. – a) Eight spherical superconducting grains located at the vertices of a cube. b) Equivalent circuital model consisting of twelve Josephson junctions and inductors located on the sides of the cubic structure.

simple network represents the circuital equivalent of a physical system consisting of eight spherical grains located at the vertices of a cube. In fig. 1 we give a schematic view of the system we are considering. Therefore, in the following section, we give a general procedure to carry out, with the aid of the Resistively Shunted Junction (RSJ) model, the equations of the motion which govern magnetic flux transitions in the system. In the third section we numerically integrate the resulting system of nonlinear differential equations in order to detect the lower threshold field in the case of external fields applied orthogonally to one of the cube faces and at $T = 0$ K. Generalization to the finite temperature case is given in the fourth section, and conclusions are drawn in the last section.

2. – The model

The circuital model in fig. 1 consists of twelve inductively coupled Josephson junctions (JJs), each junction simulating the weak superconducting coupling between two adjacent grains. The inductors are introduced to take account of the self fields generated by the circulating currents.

To each JJ, labelled according to a standard tensorial notation, we associate the gauge invariant superconducting phase difference φ_{ijk} at the contact points between grains, where the indices i, j, k take on the values 0, 1, and 2, depending on the spatial position of the junctions themselves. Let us denote the loop current circulating in a face parallel to the $(\eta\xi)$-plane, with $(\eta\xi) = (yz), (xz), (xy)$, as $I_{(\eta\xi)l}$, where l takes on the values 0 and 1. For example, the quantities $I_{(xy)0}$ and $I_{(xy)1}$ denote the loop currents seen to flow in the counterclockwise direction by an external observer in the lower and upper faces of the cube parallel to the x-y plane, respectively. The flux linked to the same face will be written as $\Phi_{(\eta\xi)l}$.

By introducing the normalized flux variable $\Psi = \Phi/\Phi_0$, and by imposing fluxoid quantization for each closed superconducting loop, we write

(1) $$2\pi\Psi_{(yz)\tilde{i}} = 2\pi n_{(yz)\tilde{i}} + \varphi_{i01} + \varphi_{i10} + \varphi_{i21} + \varphi_{i12}\,,$$

(2) $$2\pi\Psi_{(xz)\tilde{j}} = 2\pi n_{(xz)\tilde{j}} - \varphi_{0j1} + \varphi_{1j0} - \varphi_{2j1} + \varphi_{1j2}\,,$$

(3) $$2\pi\Psi_{(xy)\tilde{k}} = 2\pi n_{(xy)\tilde{k}} - \varphi_{01k} - \varphi_{10k} - \varphi_{21k} - \varphi_{12k}\,,$$

where the n's are integers, $i, j, k = 0, 2$ and $\tilde{i} = i/2$, $\tilde{j} = j/2$, $\tilde{k} = k/2$.

The total flux $\Phi_{(\eta\xi)l}$ can now be written as the sum of the induced flux and the externally applied flux $\Phi_{\text{ex}} = \mu_0 \vec{H}\cdot\vec{S}_{(\eta\xi)i}$, where $\vec{H}$ is the applied field vector and $\vec{S}_{(\eta\xi)i}$ is the area vector pertaining to the $(\eta\xi)i$ cube face, so that

(4) $$\Phi_{(\eta\xi)i} = \sum_{j=0}^{1}\sum_{\mu\nu} M_{(\eta\xi)i}^{(\mu\nu)j} I_{(\mu\nu)j} + \mu_0 \vec{H}\cdot\vec{S}_{(\eta\xi)i}\,,$$

where $M_{(\eta\xi)i}^{(\mu\nu)j}$ are the mutual inductance coefficients.

We adopt the RSJ model introducing the non-linear Josephson operator $O_{J_{ijk}}$, defined as

(5) $$O_{J_{ijk}}(\cdot) = \frac{\Phi_0}{2\pi R}\frac{\mathrm{d}}{\mathrm{d}t}(\cdot) + I_{ijk}\sin(\cdot)\,,$$

where the resistive parameter R is taken to be the same for all JJs and the quantity I_{ijk} is the maximum Josephson current of the (ijk)-junction. In this way, we can write the equations of the motion for the twelve phase variables as follows:

(6) $$O_{J_{1jk}}(\varphi_{1jk}) = I_{(xy)\tilde{k}} - I_{(xz)\tilde{j}}\,,$$

(7) $$O_{J_{i1k}}(\varphi_{i1k}) = I_{(xy)\tilde{k}} - I_{(yz)\tilde{i}}\,,$$

(8) $$O_{J_{ij1}}(\varphi_{ij1}) = I_{(xz)\tilde{j}} - I_{(yz)\tilde{i}}\,,$$

where, again, $i, j, k = 0, 2$ and $\tilde{i} = i/2$, $\tilde{j} = j/2$, $\tilde{k} = k/2$.

We notice that the currents appearing on the right hand side of eqs. (6)-(8) are implicitly defined by eq. (4). We can thus invert eq. (4) and express the flux variables in terms of the superconducting phases, in such a way that eqs. (6)-(8) become a system of coupled differential equations in the φ_{ijk} variables. In order to invert eqs. (4), however, we need to specify the mutual inductance coefficient matrix, which, for symmetry reasons, we take to be

(9) $$M_{(\eta\xi)i}^{(\mu\nu)j} = \begin{cases} \delta_{ij} L_0 - (1-\delta_{ij}) M_{\mathrm{P}}, & \text{if } (\mu\nu) = (\eta\xi)\,, \\ -M_0, & \text{if } (\mu\nu) \neq (\eta\xi)\,, \end{cases}$$

where L_0, M_0, and M_P are, respectively, the effective values of the self-inductance coefficient relative to each loop, the mutual inductance coefficient relative to mutually orthogonal faces, and the mutual inductance coefficient relative to parallel faces.

3. – Stationary magnetic states

In the present section we show the outcome of the integration process carried out on eqs. (6)-(8). In order to obtain the stationary magnetic states of the system at $T = 0$ K after ZFC ($n_{(\eta\xi)\hat{\imath}} = 0$ in eqs. (1)-(3)), we adiabatically increase the external forcing term $\Psi_{ex} = \mu_0 H S_0 / \Phi_0$, where S_0 is the area of a single face, for a system of perfectly identical JJs with maximum Josephson current I_{J0}, from zero to an arbitrary maximum value with an incremental step of $\Delta\Psi_{ex}$. The external field is taken to be parallel to the z-direction. After each increment of the forcing term, the equilibrium values of the phase variables are found, and the corresponding values of the fluxes are derived through the fluxoid quantization conditions (eqs. (1)-(3)). A standard fourth order Runge-Kutta algorithm is used, and the results are shown in fig. 2, where the fluxes linked to the cube faces are reported as a function of Ψ_{ex} for three different values of the parameter $\beta_0 = \mu_0 L_0 I_{J0} / \Phi_0$, and for the following values of the model parameters: $M_0 / L_0 = 0.2$, and $M_P / L_0 = 0.1$. The parameter β_0, in particular, can be compared to the usual SQUID parameter, and determines the ranges of reversible and

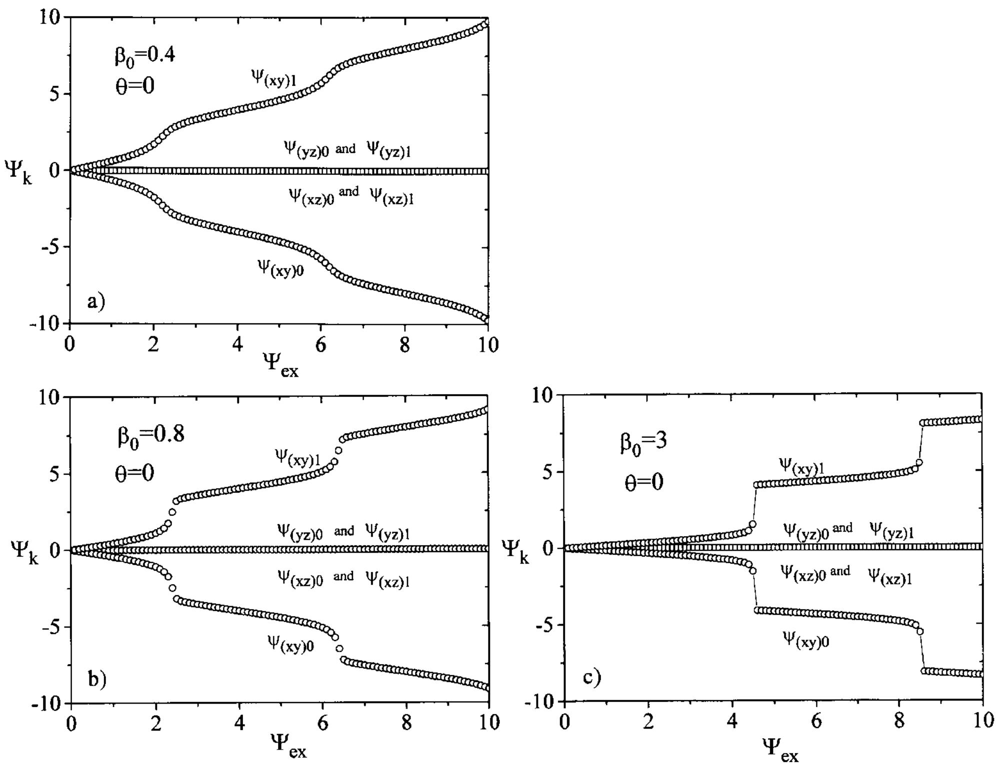

Fig. 2. – Flux linked to the six cubic faces as a function of the normalized applied flux Ψ_{ex}, for various β_0 values.

irreversible magnetic behavior of the system. As a matter of fact, as in the simple SQUID case, there exists a limiting β_0-value β_c, above which the magnetic behavior of the system goes from reversible to irreversible. Therefore, if $\beta_0 > \beta_c$, a lower threshold field H_{gc1} for irreversible flux penetration appears in the Ψ *vs.* Ψ_{ex} diagrams.

We can compute the β_c value in terms of the constants M_P and L_0. Indeed, from fig. 2, it is evident that $\Psi_{(\mu\nu)i} = 0$ if $(\mu\nu) \neq (xy)$, so that, by rewriting the equations of the motion and noticing that we can choose one independent phase, the resulting differential equation for the flux Ψ_b, linked to the upper face of the cube can be written as follows:

$$\Psi_b + \tilde{\beta} \sin(\pi \Psi_b / 2) = \Psi_{ex}, \tag{10}$$

where $\tilde{\beta} = \beta_0 (1 + M_P / L_0)$. The β_c value is obtained by taking the Ψ_b derivative of the above expression and looking at the minimum value of β_0 for which this derivative can be zero. We therefore find $\beta_c = 2/(\pi(1 + M_P/L_0))$.

From fig. 2 we can see how this crossover appears. Indeed, from fig. 2a, in which $\beta_0 < \beta_c$, no discontinuity appears in the Ψ *vs.* Ψ_{ex} curves. In fig. 2b and 2c, instead, we notice the appearance of a threshold field value for flux penetration in the *x-y* plane.

4. – Magnetic states at finite temperatures

In order to extend our analysis to the finite temperature case, we should add a white noise current term to each branch in the cubic structure. This is done by summing a stochastic variable $f_{ijk}(T)$ to the right hand side of each equation of the motion for the phase variables (eqs. (6)-(8)). By previous analyses on Josephson junction arrays, some of the authors [12] showed that the resulting behavior of the system could be obtained by modifying the characteristic intrinsically defined parameters of the model (in this case the parameter β_0). In this way, an additional extrinsic temperature dependence, coming from thermal fluctuations, is added to the intrinsic temperature dependence of β_0, essentially coming from the maximum Josephson current term I_{J0}. This leads us to consider an effective parameter $\beta_{eff}(T)$, which may take account of thermal fluctuations in the system. Given the monotonically decreasing T-dependence of I_{J0} [13], the effective parameter β_{eff} will preserve the same characteristics. On the basis of this, let us now derive an analytic expression for the threshold field value in terms of β_0, compare the analytic prediction with the numerical result, and draw information on the functional dependence of the threshold value $\Psi^1_{ex} = \mu_0 H_{gc1} S_0 / \Phi_0$ on the temperature T. By starting from eq. (10) and by setting $(d/d\Psi_b)\, \Psi_{ex} = 0$, we find, after solving simultaneausly the two equations:

$$\Psi^1_{ex} = \frac{2}{\pi} \sin^{-1}\left((1 - 1/(\pi\tilde{\beta}/2)^2)^{1/2} \right) + \tilde{\beta} (1 - 1/(\pi\tilde{\beta}/2)^2)^{1/2}. \tag{11}$$

Therefore, Ψ^1_{ex} is a monotonically increasing function of β_0, since $\tilde{\beta}$ depends linearly on β_0. It follows, from what has been previously said, that Ψ^1_{ex} is a monotonically decreasing function of T.

5. – Conclusion

We studied the stationary magnetic states of a simple three-dimensional Josephson junction network: a cube with twelve junctions. We derived the dynamical equations for flux transition for an externally applied magnetic field $\vec{H}$ in the direction orthogonal to one of the cube faces and for $T = 0$ K. We reported the fluxes linked to the cube faces as a function of the applied flux Ψ_{ex} for three different values of the parameter β_0, namely $\beta_0 = 0.4, 0.8$ and 3.0. While for $\beta_0 < \beta_c$, discontinuity appears in the Ψ *vs.* Ψ_{ex} diagrams, in the case $\beta_0 > \beta_c$, a lower threshold field H_{gc1} for irreversible flux penetration appears. Finally, we generalize our analysis to the finite T case and qualitatively discuss the temperature dependence of $\Psi^1_{ex} = \mu_0 H_{gc1} S_0 / \Phi_0$.

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