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Innovation flow through social networks: productivity distribution in France and Italy

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Abstract. From a detailed empirical analysis of the productivity of non financial firms across several countries and years we show that productivity follows a non-Gaussian distribution with 'fat tails' in the large productivity region which are well mimicked by power law behaviors. We discuss how these empirical findings can be linked to a mechanism of exchanges in a social network where firms improve their productivity by direct innovation and/or by imitation of other firm's technological and organizational solutions. The type of network-connectivity determines how fast and how efficiently information can diffuse and how quickly innovation will permeate or behaviors will be imitated. From a model for innovation flow through a complex network we show that the expectation values of the productivity of each firm are proportional to its connectivity in the network of links between firms. The comparison with the empirical distributions in France and Italy reveals that in this model, such a network must be of a scale-free type with a power-law degree distribution in the large connectivity range.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management – 89.75.Hc Networks and genealogical trees – 89.75.-k Complex systems – 89.75.Da Systems obeying scaling laws

1 Introduction

The recent availability of huge sets of longitudinal firm-level data has generated a number of productivity studies in the economic literature [1–7]. There are several measures of productivity [8]. In this work we consider two basic measures: labour and capital productivity. The labour productivity is defined as value added over the amount of employees (where value added, defined according to standard balance sheet reporting, is the difference between total revenue and cost of input excluding the cost of labour). Although elementary, this measure has the advantage of being accurately approximated from the available data. The other alternative measure is the capital productivity which is defined as the ratio between value added and fixed assets (i.e. capital). This second measure has some weakness since the firms' assets change continuously in time (consider for instance the value associated with the stock price). Usually the literature recognizes that the productivity distribution is not normally distributed [7], and empirically 'fat tails' with power law behaviors are observed. But the mainstream proposed explanations cannot retrieve this power law tails yielding — at best — to log-normal distributions [9,10]. In this work we approach this problem from a different perspective by analyzing the effect that interactions between firms have on the productivity. According to the evolutionary perspective [11,12], firms improve their productivity implementing new technological and organizational solutions and, in this way, upgrade their routines. The search for more efficient technologies is carried out in two ways: (1) by *innovation* (direct search for more efficient routines); (2) by *imitation* of the most innovative firms [13,14]. In practice, one can figure out that once new ideas or innovative solutions are conceived by a given firm then they will percolate outside the firm that originally generated them by imitation from other firms. In this way the innovation flows through the network of contacts and communications between firms. Therefore, such a network must play a decisive role in the process. Our approach mimics such a dynamics by considering a simple type of interaction but assuming that they take place through a complex network of contacts. The challenge here is to understand whether such a network of contacts could lead to the emergence of 'fat tails' in the productivity distributions.

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In this paper we address this problem by *first* proposing a model for the production and flow of innovation; *second* by performing an empirical analysis for France and Italy (based on the data set *Amadeus*, which records data of over 6 million European firms from 1990 to 2002 [15]); *third* by comparing the analytical results with the empirical ones showing that indeed power law tails can emerge from scale-free contact-networks and observing a good agreement between the theoretical predictions and the empirical findings.

The paper is organized as follows: Section 2 recalls the concept of social network; Section 3 introduces the model supporting the technological distribution, Section 4 describes some empirical findings while in Section 5 we compare these empirical findings with the theoretical prediction. A concluding section summarizes the main results in the light of recent developments in the study of social networks.

2 Contact networks in social systems

Systems constituted of many elements can be naturally associated with networks linking interacting constituents. Examples in natural and artificial systems are: food webs, ecosystems, protein domains, Internet, power grids. In social systems, networks also emerge from the linkage of people or group of people with some pattern of contacts or interactions. Examples are: friendships between individuals, business relationships between companies, citations of scientific papers, intermarriages between families, sexual contacts. The relevance of the underlying connection-network arises when the collective dynamics of these systems is considered. Recently, the discovery that, above a certain degree of complexity, natural, artificial and social systems are typically characterized by networks with power-law distributions in the number of links per node (degree distribution), has attracted a great deal of scientific interest [16–18]. Such networks are commonly referred as scale-free networks and have degree distribution: $p_k \sim k^{-\alpha}$ (with p_k the probability that a vertex in the network chosen uniformly at random has degree k). In scale-free networks most nodes have only a small number of links, but a significant number of nodes have a large number of links, and all frequencies of links in between these extremes are represented. The earliest published example of a scale-free network is probably the study of Price [19] for the network of citations between scientific papers. Price found that the exponent α has value 2.5 (later he reported a more accurate figure of $\alpha = 3.04$). More recently, power law degree distributions have been observed in several networks, including other citation networks, the World Wide Web, the Internet, metabolic networks, telephone calls and the networks of human sexual contacts [17,18,20–22]. All these systems have values of the exponents α in a range between 0.66 and 4, with most occurrences between 2 and 3 [23–26].

When analyzing industrial dynamics, it is quite natural to consider the firms as interacting within a network of contacts and communications. In particular, when the productivity is considered, such a network is the structure through which firms can imitate each-other.

3 Innovation flow

In this section we introduce a model for the flow of innovation through the system of firms. The general idea is that the innovation, originally introduced in a given firm 'i' at a certain time t, can spread by imitation across the network of contacts between firms. In this way, interactions force agents to progressively adapt to an ever changing environment.

Let us start by describing such a production and flow of innovation by means of the following equation which describes the evolution in time of the productivity x_l of a given firm 'l':

$$x_l(t+1) = x_l(t) + A_l(t) + \sum_{j \in \mathcal{I}_l} Q_{j\to l}(t)[x_j(t) - x_j(t-1)] - \sum_{\tau=1}^{t-1} q_l^{(\tau)}(t)[x_l(t-\tau) - x_l(t-\tau-1)]. \quad (1)$$

The term $A_l(t)$ is a stochastic additive quantity which accounts the progresses in productivity due to innovation. The terms $Q_{j\to l}$ are instead exchange factors which model the imitation between firms. These terms take into account the improvement of the productivity of the firm 'l' in consequence of the imitation of the processes and innovations that had improved the productivity of the firm 'j' at a previous time. Such coefficients are in general smaller than one because the firms tend to protect their innovation content and therefore the imitation is in general incomplete. In the following we will consider only the static cases where these quantity are independent on t. The term $q_l^{(\tau)}$ is:

$$q_l^{(1)} = \sum_{j \in \mathcal{I}_l} Q_{j\to l} Q_{l\to j} \text{ for } \tau = 1 \quad (2)$$

$$q_l^{(\tau)} = \sum_{j \in \mathcal{I}_l} Q_{j\to l} \times \sum_{h_1 \ldots h_{\tau-1}} Q_{l\to h_1} Q_{h_1\to h_2} \ldots Q_{h_{\tau-1}\to j} \text{ for } \tau \geq 2. \quad (3)$$

This term excludes back-propagation: firm 'l' imitates only improvements of the productivity of firm 'j' which have not been originated by imitation of improvements occurred at the firm 'l' itself at some previous time. The system described by equation (1) can be viewed as a system of self-avoiding random walkers with sources and traps.

The probability $P_{t+1}(y,l)dy$ that the firm l at the time $t+1$ has a productivity between y and $y+dy$ is related to the probabilities to have a set $\{Q_{j\to l}\}$ of interaction coefficients and a set of additive coefficients $\{A_l(t)\}$ such that a given distribution of productivity $\{x_j(t)\}$ at the time t yields, through equation (1), to the quantity y for the agent l at time $t+1$. This is:

$$P_{t+1}(y,l) = \int_{-\infty}^{\infty} da \Lambda_t(a,l) \prod_{\xi=0}^{t-1} \int_{-\infty}^{\infty} dx_1^{(\xi)} P_{t-\xi}\left(x_1^{(\xi)},1\right)\cdots \times \int_{-\infty}^{\infty} dx_N^{(\xi)} P_{t-\xi}\left(x_N^{(\xi)},N\right) \times \delta\left(y - a - x_l^{(0)} - \sum_{j\in\mathcal{I}_l}\left[x_j^{(0)} - x_j^{(1)}\right] Q_{j\to l} + \sum_{\tau=l}^{t-1} q_l^{(\tau)}\left[x_l^{(\tau)} - x_l^{(\tau+1)}\right]\right), \quad (4)$$

where $\delta(y)$ is the Dirac delta function and $\Lambda_t(a,l)$ is the probability density to have at time t on site l an additive coefficient $A_l(t) = a$. Let us introduce the Fourier transformation of $P_t(y,l)$ and its inverse

$$\hat{P}_t(\varphi,l) = \int_{-\infty}^{\infty} dy e^{+iy\varphi} P_t(y,l)$$
$$P_t(y,l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\varphi e^{-iy\varphi}\hat{P}_t(\varphi,l). \quad (5)$$

In Appendix A, we show that equation (4) can be rewritten in term of these transformations, resulting in:

$$\hat{P}_{t+1}(\varphi,l) = \hat{\Lambda}_t(\varphi,l)\hat{P}_t(\varphi,l)\prod_{\xi=2}^{t-1}\hat{P}_{t-\xi}\left((-q_l^{(\xi)}+q_l^{(\xi-1)})\varphi,l\right) \times \hat{P}_0\left(q_l^{(t-1)}\varphi,l\right)\hat{P}_{t-1}\left(-q_l^{(1)}\varphi,l\right) \times \prod_{j\in\mathcal{I}_l}\hat{P}_t\left(Q_{j\to l}\varphi,j\right)\hat{P}_{t-1}(-Q_{j\to l}\varphi,j), \quad (6)$$

with $\hat{\Lambda}_t(\varphi,l)$ being the Fourier transform of $\Lambda_t(a,l)$. From this equation we can construct a relation for the propagation of the cumulants of the productivity distribution. Indeed, by definition the cumulants of a probability distribution are given by the expression:

$$k_l^{(\nu)}(t) = (-i)^\nu \frac{d^\nu}{d\varphi^\nu}\ln\hat{P}_t(\varphi,l)\Big|_{\varphi=0}, \quad (7)$$

where the first cumulant $k_l^{(1)}(t)$ is the expectation value of the stochastic variable x_l at the time t ($\langle x_l(t)\rangle$) and the second cumulant $k_l^{(2)}(t)$ is its variance $\left(\sigma_l^2(t)\right)$. By taking the logarithm of equation (6) and applying equation (7) we get:

$$k_l^{(\nu)}(t+1) = c^{(\nu)}(t) + k_l^{(\nu)}(t) + \sum_{\xi=2}^{t-1}\left(q_l^{(\xi-1)} - q_l^{(\xi)}\right)^\nu k_l^{(\nu)}(t-\xi) + \left(q_l^{(t-1)}\right)^\nu k_l^{(\nu)}(0) + \left(-q_l^{(1)}\right)^\nu k_l^{(\nu)}(t-1) + \sum_{j\in\mathcal{I}_l}\left[(Q_{j\to l})^\nu k_j^{(\nu)}(t) + (-Q_{j\to l})^\nu k_j^{(\nu)}(t-1)\right]. \quad (8)$$

In reference [27], Maddison shows that the average innovation rate of change in the OECD countries since 1870 has been roughly constant. In our formalism this implies

$$\frac{\langle A_l(t+1)\rangle - \langle A_l(t)\rangle}{\langle A_l(t)\rangle} \sim \text{const.} \quad (9)$$

Therefore, the mean of the additive term in equation (1) ($\langle A_l(t)\rangle$) must grow exponentially with time and consequently the first cumulant (the average indeed) reads: $c^{(1)} = c_0^{(1)}\left(c_1^{(1)}\right)^t$. Equivalently we assume an exponential growth also for the other moments $\left(c^{(\nu)} = c_0^{(1)}\left(c_1^{(\nu)}\right)^t\right)$.

Equation (8) can now be solved by using a mean-field, self-consistent solution (neglecting correlations and fluctuations in the interacting firms) obtaining:

$$k_l^{(1)}(t) = \frac{1}{A}\frac{c_0^{(1)}c_1^{(1)}}{\left(c_1^{(1)}-1\right)}\left[1+\bar{a}Qz_l\right]\left(c_1^{(1)}\right)^t \text{ for } \nu=1$$
$$k_l^{(\nu)}(t) = \frac{c_0^{(\nu)}}{B_\nu}\left[1+\left(1+\frac{(-1)^\nu}{c_1^{(\nu)}}\right)\bar{b}^{(\nu)}Q^\nu z_l\right]\left(c_1^{(\nu)}\right)^t \text{ for } \nu>1 \quad (10)$$

where

$$\bar{a} = \frac{1}{1-\left\langle\frac{Qz_l}{A}\right\rangle}\frac{1}{\langle A\rangle} \quad (11)$$

$$\bar{b}^{(\nu)} = \frac{1}{1+\left\langle\frac{(1+(-1)^\nu/c_1^{(\nu)})Q^\nu z_l}{B_\nu}\right\rangle}\frac{1}{\langle B_\nu\rangle} \quad (12)$$

and

$$A = c_1^{(1)} + z_l\sum_{\xi=1}^{t-1}\frac{Q^{\xi+1}}{\left(c_1^{(1)}\right)^\xi} \quad (13)$$

$$B_\nu = -1 + c_1^{(\nu)} - z_l^\nu\left[\frac{(-Q^2)^\nu}{c_1^{(\nu)}} + \sum_{\xi=2}^{t-1}\frac{\left(Q^\xi - Q^{\xi+1}\right)^\nu}{\left(c_1^{(\nu)}\right)^\xi} + \frac{(Q^t)^\nu}{\left(c_1^{(\nu)}\right)^t}\right] \quad (14)$$

with Q being the average exchange factor. When this exchange term is small, equation (10) can be highly simplified by taking the first order in Q only, leading to:

$$k_l^{(1)}(t) \sim \frac{c_0^{(1)}}{c_1^{(1)}-1}\left[1+z_l\frac{Q}{c_1^{(1)}}\right]\left(c_1^{(1)}\right)^t$$
$$k_l^{(\nu)}(t) \sim \frac{c_0^{(\nu)}}{c_1^{(\nu)}-1}\left(c_1^{(\nu)}\right)^t. \quad (15)$$

Equation (10) (and its simplified form, Eq. (15)) describes a mean productivity which grows at the same rate

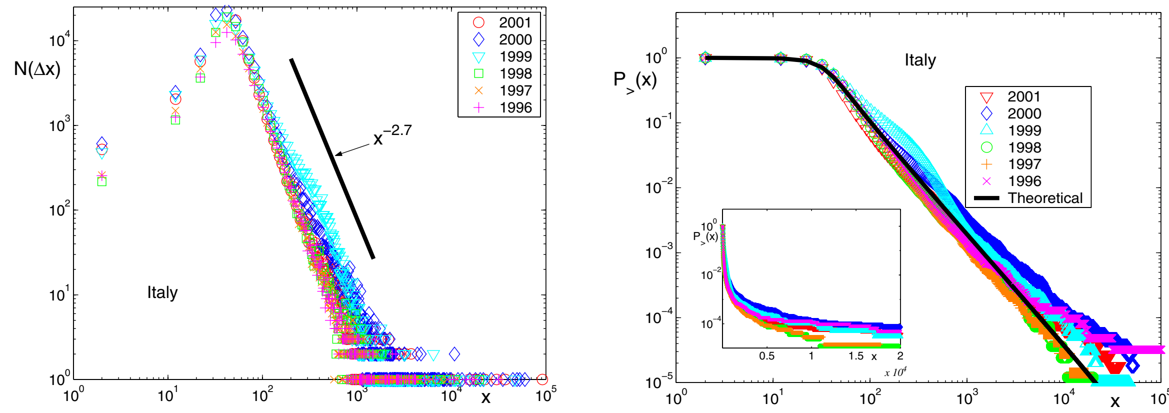

Fig. 1. Frequency distributions (left) and complementary cumulative distributions (right) for the labour productivity in Italy in the years 1996–2001. The theoretical behavior is for $\alpha = 2.7$, $m = 22$, $n = 11$, $\sigma = 10$ and $\beta = 3$. The insert shows $P_>(x)$ vs. x in log-normal scale.

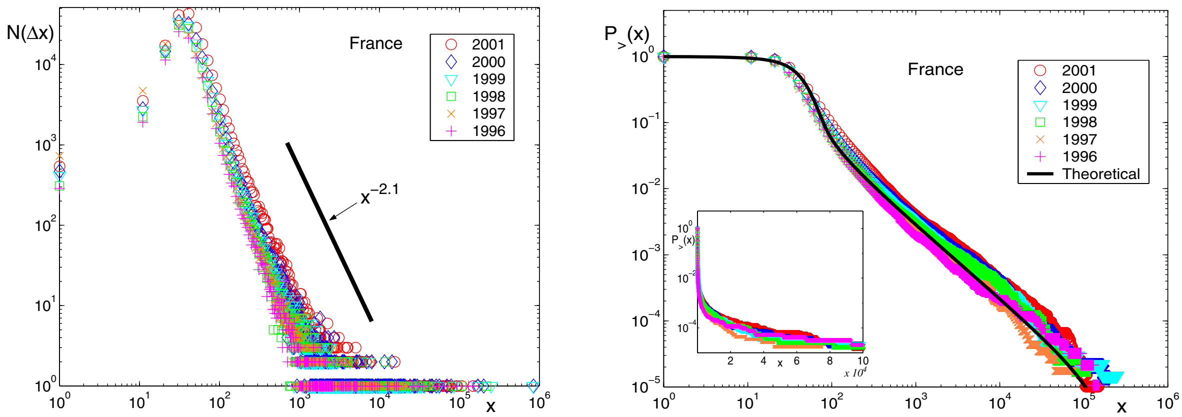

Fig. 2. Frequency distributions (left) and complementary cumulative distributions (right) for the labour productivity in France in the years 1996–2001. The theoretical behavior is for $\alpha = 2.1$, $m = 30$, $n = 4$, $\sigma = 20$ and $\beta = 1$. The insert shows $P_>(x)$ vs. x in log-normal scale.

of the mean innovation growth (as a power of $c_1^{(1)}$) and is directly proportional to the number of connections that the firm has in the exchange network. Equation (10) also shows that all the cumulants increase with a corresponding power rate $\left((c_1^{(\nu)})^t\right)$. But, if we analyze the *normalized cumulants*: $\lambda^{(\nu)}(t) = k_l^{(\nu)}(t)/\left[k_l^{(2)}(t)\right]^{\nu/2}$ we immediately see that at large t they all tend to zero excepted for the mean and the variance. Therefore the probability distributions tend to Gaussians at large times.

Summarizing, in this section we have shown that, at large t, the expectation value of the productivity level of a given firm is proportional to its connectivity in the network of interaction and the fluctuations around this expectation-value are normally distributed. Each firm has a different connectivity and therefore the aggregate probability distribution for the productivity of the ensemble of firms is given by a normalized sum of Gaussians with averages distributed according with the network connectivity.

4 Empirical analysis

Figures 1–4 show the log–log plot of the frequency distributions (left) and the complementary cumulative distributions (right) of labour productivity and capital productivity measured as quotas of total added value of the firms. In these figures the different data sets correspond to the years 1996–2001 for two different countries: France and Italy. The frequency distributions show a very clear non-Gaussian character: they are skewed with asymmetric tails and the productivity (Figs. 1–4 (left)) exhibits a leptokurtic peak around the mode with 'fat tails' (for large productivities) which show a rather linear behavior in a log–log scale. In these figures we also report, for comparison, the linear trend corresponding to power-law distributions ($N(x) \propto x^{-\alpha}$ with $\alpha = 2.7, 2.1, 3.8$ and 4.6 respectively in Figs. 1–4). The complementary cumulative distributions ($P_>(x)$, being the probability to find a firm with productivity larger than x) also show a linear trend at large x (in log–log scale) implying a non-Gaussian character with the probability for large productivities well

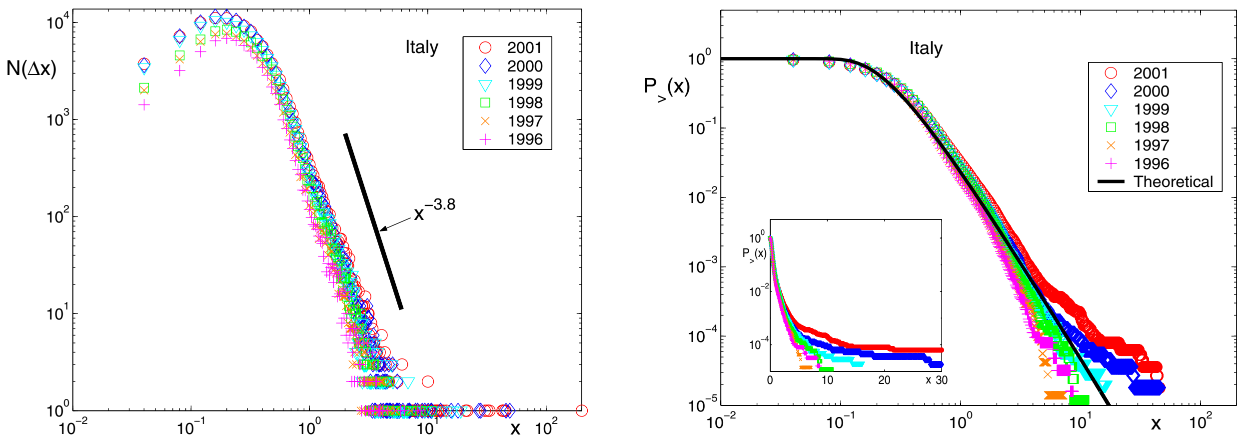

Fig. 3. Frequency distributions (left) and complementary cumulative distributions (right) for the capital productivity in Italy in the years 1996–2001. The theoretical behavior is for $\alpha = 3.8$, $m = 0.04$, $n = 0.02$, $\sigma = 0.01$ and $\beta = 25$. The insert shows $P_>(x)$ vs. x in log-normal scale.

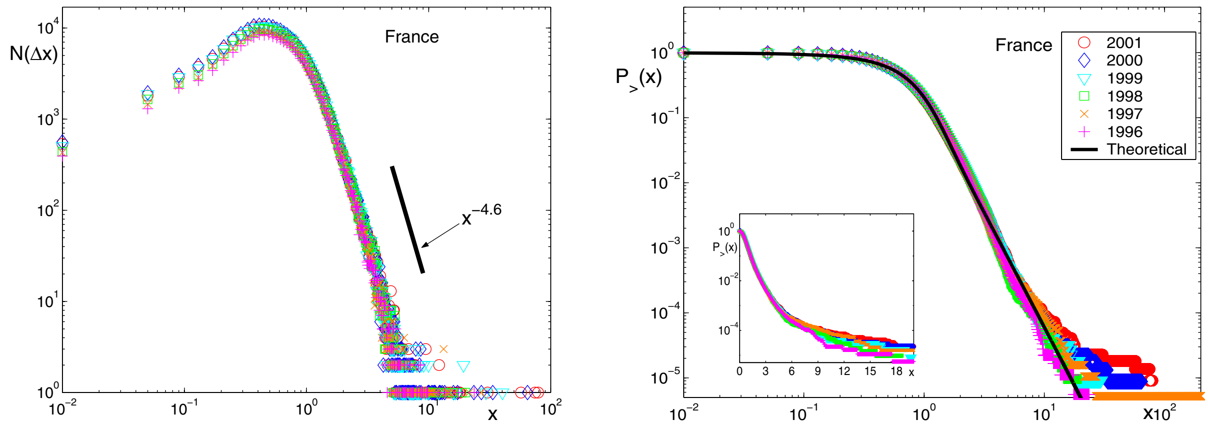

Fig. 4. Frequency distributions (left) and complementary cumulative distributions (right) for the capital productivity in France in the years 1996–2001. The theoretical behavior is for $\alpha = 4.6$, $m = 0.06$, $n = 0.02$, $\sigma = 0.4$ and $\beta = 68$. The insert shows $P_>(x)$ vs. x in log-normal scale.

mimicked by a power-law behavior. The 'fat tails' character of such distributions is highlighted in the inserts of Figures 1–4 (right) where log-normal plots show that the decay of $P_>(x)$ with x is much slower than exponential.

5 Comparison with theory

The model presented in this paper predicts that the aggregate distribution for the productivity of the ensemble of firms is given by a normalized sum of Gaussians with averages distributed according with the connectivity in the network of contacts/interactions among firms. Therefore, it is the underlying network which shapes the aggregate productivity distributions. Empirically we observe the occurrence of 'fat tails' in the productivity distributions. Accordingly with the present model, such slow decaying distributions must be the "consequence" [41] of special structure of the contact/information network which must also has slow decaying tails in its degree distribution being therefore of "scale-free" type. Indeed, as discussed in the Section 2, power-law-tailed degree distributions are very common in many social and artificial networks [28–33]. It is therefore natural to find that the contact/information network through which firms can exchange and imitate technological innovations must also have a degree distribution characterized by a power law in the large connection-numbers region. Recent works [34] support this view finding power-law distributions in the firms' networks.

Comparisons between the theoretical predictions (Eq. (15)) and the empirical findings are shown in the Figures 1–4 (right). We find a quantitatively rather good agreement by considering an underlying scale-free network with degree distribution given by $p_k \propto k^{-\alpha} \exp(-\beta/k)$ and, accordingly with equation (15), $k_l^{(1)} = m + z_l n$ and a variance equal to σ. We note that, although there are several parameters, the behavior for large productivity is controlled only by the power-law exponent $-\alpha$. On the other hand, in the small and the middle range of the distribution the other parameters have a larger influence.

From our analysis we observe that the theoretical curves fit the empirical findings well by assuming the power law exponent equal to $\alpha = 2.7$ and 2.1 for the labour productivity in Italy and France respectively. These exponents are in good agreement with the typical degree distribution in social networks. On the other hand the capital productivity presents much steeper decays which can be fitted with exponents 3.8 and 4.6 respectively. However the very high capital productivity regions show a slowing down which could be fitted with lower exponents.

6 Conclusions and perspectives

In this paper we have shown that the productivity of non-financial firms is: (i) fat tailed with a slow decrease in the large productivity region which is well mimicked by a power law behavior; (ii) this result is robust to different measures of productivity (added value-capital and capital-labor ratios); and (iii) it is persistent over time (from 1996 to 2001) and countries (France and Italy). We have also shown that the empirical evidence corroborates the prescription of the evolutionary approach to technical change and demonstrated that power law distributions in productivity can be linked to a simple mechanism of exchanges within a social network. In particular, we have presented a simple model for innovation flow through the network of contacts between the firms showing that the expectation values of the productivity level of each firm are proportional to the connectivity of the network. The comparison with the empirical data for France and Italy indicates that such a network must be of a scale-free type with a power-law degree distribution in the large connection-numbers region. Indeed, this yields to productivity distributions which are in good quantitative agreement with empirical data showing power-law kind tails characterized by the same exponent of the degree distribution [35]. We must stress that in the present formulation we have assumed the simplest kind of interaction in an underlying network which is fixed in time. This allows obtaining equilibrium solutions. On the other hand, a more realistic analysis should consider a non-static underlying network and a non-linear type interaction which will therefore lead to non-equilibrium trajectories modulated by the fluctuation in the underlying network.

Recent developments in social network theory show that the network of connections is a dynamical structure where performative ties between non-previously linked nodes can be created [37], where the absorption of information is related to the previous knowledge [38] and where innovation flow might follow different behaviors [39] and strategies [40]. All these effects can be included in the present model by considering the exchange coefficients $Q_{j\to l}$ between two firms as dynamical variables which vary with the time (performative ties [37]), which are functions of the level of productivity of the firms (absorptive capacity [38] and strategy [40]) and which depend on the history of the productivity in each firm (non instantaneous adsorption of organizational practices [39]). However, the aim of the present work is to show the specific influence of a static network of simple interactions on the productivity distribution. We expect that the results obtained for such a static network will hold also for a 'quasi-static' network where the time-scales of the variations in the network structure are much slower than the characteristic time of equilibrium in the exchanges of productivity. The crossover between such a 'quasi-static' system to a (non-equilibrium) dynamical system is left to future research. In this paper we had a narrower goal: to show that empirical evidence is very well fitted by the evolutionary view of technical change.

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Appendix A: Cumulant propagation

By using the Fourier transformation (Eq. (5)), equation (4) becomes:

$$
\begin{aligned}
&P_{t+1}(y,l) = \\
&\int_{-\infty}^{\infty} da \Bigg\{ \Lambda_t(a,l) \prod_{\xi=0}^{t-1} \Bigg[\frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} dx_1^{(\xi)} \cdots \int_{-\infty}^{\infty} dx_N^{(\xi)} \\
&\times \int_{-\infty}^{\infty} d\varphi_1^{(\xi)} e^{-ix_1^{(\xi)}\varphi_1^{(\xi)}} \hat{P}_{t-\xi}\left(\varphi_1^{(\xi)}, 1\right) \cdots \\
&\times \int_{-\infty}^{\infty} d\varphi_N^{(\xi)} e^{-ix_N^{(\xi)}\varphi_N^{(\xi)}} \hat{P}_{t-\xi}\left(\varphi_N^{(\xi)}, N\right) \Bigg] \int_{-\infty}^{\infty} d\phi \\
&\times \frac{1}{2\pi} e^{-i\left(y-a-x_l^{(0)}-\sum_{j\in\mathcal{I}_l}\left[x_j^{(0)}-x_j^{(1)}\right] Q_{j\to l}+\sum_{\tau=l}^{t-1} q_l^{(\tau)}\left[x_l^{(\tau)}-x_l^{(\tau+1)}\right]\right)\phi} \Bigg\},
\end{aligned}
\tag{A.1}
$$

where the Dirac delta function has been written as

$$
\delta(y-y_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi e^{-i(y-y_0)\phi}. \tag{A.2}
$$

Equation (A.1) can be re-written as:

$$P_{t+1}(y,l) = \frac{1}{(2\pi)}\int_{-\infty}^{\infty} da \Big\{ \Lambda_t(a,l) \int_{-\infty}^{\infty} d\phi e^{-i(y-a)\phi}$$

$$\times \prod_{\xi=0}^{t-1}\left[\frac{1}{(2\pi)^N}\int_{-\infty}^{\infty} d\varphi_l^{(\xi)} \left(\hat{P}_{t-\xi}(\varphi_l^{(\xi)},l)\right.\right.$$

$$\times \int_{-\infty}^{\infty} dx_l^{(\xi)} e^{-i\left(\varphi_l^{(0)}-\phi\right)x_l^{(0)}} e^{-i\sum_{\tau=2}^{t-1}\left(\varphi_l^{(\tau)}+q_l^{(\tau)}\phi-q_l^{(\tau-1)}\phi\right)x_l^{(\tau)}}$$

$$\left. e^{-i\left(\varphi_l^{(t)}-q_l^{(t-1)}\phi\right)x_l^{(t)}} e^{-i\left(\varphi_l^{(1)}-q_l^{(1)}\phi\right)x_l^{(1)}}\right)$$

$$\times \prod_{j\in\mathcal{I}_l}\int_{-\infty}^{\infty} d\varphi_j^{(\xi)}\left(\hat{P}_{t-\xi}\left(\varphi_j^{(\xi)},j\right)\right.$$

$$\left.\left.\times \int_{-\infty}^{\infty} dx_j^{(\xi)} e^{-i\left[\left(\varphi_j^{(0)}-Q_{j\to l}\phi\right)x_j^{(0)}+\left(\varphi_j^{(1)}+Q_{j\to l}\phi\right)x_j^{(1)}\right]}\right)\right]\Big\}. \tag{A.3}$$

The integration over the x's yields

$$P_{t+1}(y,l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} da \Big\{ \Lambda_t(a,l)\int_{-\infty}^{\infty} d\phi \Big[e^{-i(y-a)\phi}\hat{P}_t(\phi,l)$$

$$\times \prod_{\xi=2}^{t-1}\hat{P}_{t-\xi}\left(\left(-q_l^{(\xi)}+q_l^{(\xi-1)}\right)\phi,l\right)\hat{P}_0\left(q_q^{(t-1)}\phi,l\right)$$

$$\times \hat{P}_{t-1}\left(-q_l^{(1)}\phi,l\right)\prod_{j\in\mathcal{I}_l}\hat{P}_t(Q_{j\to l}\phi,j)\hat{P}_{t-1}(-Q_{j\to l}\phi,j)\Big]\Big\}. \tag{A.4}$$

Its Fourier transform is:

$$\hat{P}_{t+1}(\varphi,l) =$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} da \left\{ \Lambda_t(a,l)\int_{-\infty}^{\infty} d\phi \left[e^{ia\phi}\int_{-\infty}^{\infty} dy e^{-iy(\phi-\varphi)}\right.\right.$$

$$\times \hat{P}_t(\phi,l)\prod_{\xi=2}^{t-1}\hat{P}_{t-\xi}\left((-q_l^{(\xi)}+q_l^{(\xi-1)})\phi,l\right)$$

$$\left.\times \hat{P}_0\left(q_q^{(t-1)}\phi,l\right)\hat{P}_{t-1}\left(-q_l^{(1)}\phi,l\right)\right]$$

$$\left.\times \prod_{j\in\mathcal{I}_l}\hat{P}_t(Q_{j\to l}\phi,j)\hat{P}_{t-1}(-Q_{j\to l}\phi,j)\right\}. \tag{A.5}$$

Equation (A.5) can be integrated over y giving the Fourier transform of equation (4) which is equation (6) in Section 3.

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