MOST 2014 – Mathematics of String Theory

Dates: June 2-3,2014
Place: King's College London
Web: http://www.mth.kcl.ac.uk/∼ss299/MOST

Abstracts

Tom Bridgeland (Sheffield): *Geometry of spaces of stability conditions*

I will start by giving the usual string-theoretic motivation for stability conditions following the original ideas of Mike Douglas. The meat of the talk will be about joint work with Tom Sutherland and Qiu Yu computing spaces of stability conditions for the d-dimensional Calabi-Yau category associated to the A_2 quiver (for all d). As predicted by mirror symmetry our results relate closely to the Frobenius structure on the unfolding space of the singularity x^3=0. If time allows I will report on my never-ending failure to give a categorical construction of this Frobenius structure.

Joe Fine (Brussels): *A gauge theoretic approach to Einstein 4-manifolds*

I will explain a new way to formulate Einsteins equations in dimension 4, using the language of gauge theory, the result of joint work with Dmitri Panov and Kirill Krasnov. The fundamental objects are no longer Riemannian metrics, but instead certain connections over a 4-manifold M. A connection defines a metric via its curvature, analogous to the relationship between the electromagnetic potential and field in Maxwell's theory. The total volume is an action for the theory, whose critical points give Einstein metrics. At the same time, the connection also determines a symplectic structure on an associated 6-manifold Z which fibres over M. The connections involved in this story come in two flavours: positive and negative. When they give Einstein metrics this sign agrees with that of the Einstein constant. The sign also determines the first Chern class of Z: positive leads to symplectic Fanos whilst negative leads to symplectic Calabi-Yaus. I will close with some open questions concerning the symplectic manifolds which arise this way.

Xenia de la Ossa (Oxford): *The Moduli Space of N=1 Supersymmetric Heterotic String Theories*

I will begin by introducing the geometry of heterotic string theories compactified on a six dimensional Riemannian manifold X, with a polystable holomorphic bundle V on X. The equations constraining the geometry of the pairs (X, V) are known as as the Strominger system. I will then describe my joint work with Eirik Svanes on the moduli space of these geometries. I will show that the infinitesimal deformations of the Strominger system are equivalent to the tangent space to the moduli space of a holomorphic structure $\bar{D}$ on a bundle Q constructed by extending bundle $E = \mathrm{End}(V) \oplus \mathrm{End}(TX) \oplus TX$ by the cotangent space T^*X, with extension class given by a map $\mathcal{H}$ which enforces the anomaly cancelation

condition for the 3-form field H. Time permitting, I will discuss my work with P Candelas and J McOrist on the existence of a Kähler metric on the moduli space as predicted by the fact that the effective 4-dimensional theory must preserve at least one supersymmetry. I will try to explain why these results, and the problems which remain unsolved in the context of heterotic string theories, should be of interest to mathematicians.

Tudor Dimofte (IAS): *3d N=4 theories, symplectic duality, and knot homology*

Categories of branes on hyperkahler manifolds (such as Nakajima quiver varieties) have recently been used in mathematics to provide geometric constructions of knot homologies. I will propose a concrete realization of these categories via 3d N=4 gauge theories and sigma models, explain how 3d mirror symmetry leads to the mathematical phenomenon of "symplectic duality," and relate the 3d N=4 theories to 2d Landau-Ginzburg models that appear in the physical constructions of knot homology.

Mark Gross (Cambridge): *Mirror symmetry and cluster algebras*

I will talk about recent work on cluster algebras, joint with Paul Hacking, Sean Keel and Maxim Kontsevich. Cluster algebras arose from the combinatorics of Lusztig and Kashiwara's canonical bases, and since then have found wide applications, including in string theory. Using techniques from mirror symmetry, we prove a number of the significant conjectures in the area, including positivity of the Laurent phenomenon.

Lotte Hollands (Oxford): *Spectral networks, abelianization and SL(3,C) Fenchel-Nielsen coordinates*

Spectral networks are certain networks of trajectories on a Riemann surface C, which arise naturally in four-dimensional N=2 theories. They do not only provide a useful tool to compute BPS particle spectra of such theories, but also have a more mathematical application: given a spectral network one can abelianize flat SL(K,C) connections on the surface C. This construction inherently produces a Darboux coordinate system on the moduli space of flat SL(K,C) connections. In this talk I will describe my work with Andy Neitzke on how to retrieve the complex Fenchel-Nielsen coordinates from a particular type of spectral network. This inspires a generalization of the notion of Fenchel-Nielsen coordinates to moduli spaces of higher rank flat connections. I will emphasize the importance of this construction for the so-called T3 theory, and explain its relevance in computing partition functions of four-dimensional N=2 theories.

Sean Keel (UT Austin): *An Intrinsic Global Positioning System for Calabi-Yau manifolds*

Gross, Hacking and I conjectured that the vector space of regular functions on an affine manifold with a holomorphic volume form (of the right sort) comes with a canonical basis. I'll explain our partial results, and some of the many applications to representation theory, Teichmuller theory, Mori theory, string theory, cluster algebras and mirror symmetry. I won't assume any prior knowledge of any of these topics – if you know what is meant by the order

of pole of a meromorphic function along a codimension one submanifold you know enough to absorb the main points.

David Skinner (Cambridge): *Quadratic Differentials and Null Geodesics*

TBA

Richard Thomas (Imperial): *The Katz-Klemm-Vafa formula*

I will give an introduction to Gromov-Witten theory and stable pairs, and their relationship via the MNOP conjecture. Then I will outline an application – a proof of the "KKV conjecture", expressing the Gromov-Witten invariants of K3 surfaces in all classes and genera (and all multiple covers etc) in terms of modular forms. This is joint work with Rahul Pandharipande.

Short Talks

Monday	4.30-4.50pm	Larfors
	4.50-5.10pm	Lawrie
	5.10-5.30pm	Tonkonog
Tuesday	4.30-4.50pm	Schulgin
	4.50-5.10pm	Abouaf
	5.10-5.30pm	Assel

Roland Abuaf (Imperial): *Hyperkahler categories*

Homological Mirror Symmetry has been proposed by Kontsevich 20 years ago ans is still a driving conjecture in mathematical string theory. In the special case of Hyperkahler manifolds, Homological Mirror Symmetry predicts a number of autoequivalences of the derived categories of such varieties which do not come from automorphisms of the complex structure.

These autoequivalences have been proved to exist for many hyperkahler manifolds. However, the very limited number of hyperkahler varieties we know makes it difficult to understand if these observations are significant or not. Hence, it seems very important to have more examples of derived categories of such varieties. Or perhaps not so much derived categories of hyperkahler manifolds as such, but at least examples of triangulated categories which closely look like them.

In this talk, I will introduce the notion of "Hyperkahler Categories". I will explain some of their basic properties and show how Kuznetov's theory of Categorical Crepant Resolution of Singularities enables me to produce new examples of hyperkahler categories. I will in particular focus on an exotic 4 dimensional example for which I can prove the existence of P^2-twists la Huybrechts-Thomas. This confirms some implications of Homological Mirror Symmetry in the non-commutative setting.

Benjamin Assel (King's): *Supersymmetric localisation on four-manifolds*

I will present recent (/ongoing) developments in computing the exact partition function of four-dimensional gauge theories with N=1 supersymmetry on curved manifolds, using the techniques of supersymmetric localisation. The emphasis will be placed on results valid for arbitrary four-manifolds admitting Killing spinors. The results of the localisation computation on Hopf surfaces with S1xS3 topology will be presented as an example.

Magdalena Larfors (Oxford): *SU(3) structure manifolds and heterotic domain wall solutions*

In the absence of background fluxes and sources, compactifying string theories on Calabi-Yau three-folds leads to supersymmetric solutions. Turning on fluxes, e.g. to lift the moduli of the compactification, generically forces the three-fold to break the Calabi-Yau conditions, and instead fulfill the weaker geometrical condition of having a reduced structure group. Many mathematical properties of these spaces remain unknown. In this talk I will discuss some features of the moduli spaces of manifolds with SU(3) structure from the perspective of heterotic domain wall compactifications.

Craig Lawrie (King's): *Box Graphs and Singular Fibers*

We study crepant resolutions of singular elliptically fibered Calabi-Yau fourfolds by analysing the phases of the Coulomb branch of the 3d N=2 gauge theories arising from the compactification of M-theory on the Calabi-Yau. Each phase corresponds to a particular resolution of the singular fourfold and has a description in terms of a 'decorated box graph', or a path on such a graph. The phases are related by deformations of this path, called flops, which correspond to geometric flop transitions between the resolved geometries. Using these techniques we can determine all the resolved geometries and the transitions between them, and we can enumerate the fiber types in codimensions 2 and 3, including new, non-Kodaira, fiber types.

Waldemar Schulgin (Brussels): *The Diffeomorphism Algebra on the Worldsheet*

We associate worldsheet vertex operators to space-time diffeomorphisms in flat space string theory, and compute their algebra, which is a diffeomorphism algebra with higher derivative corrections. As an application, we realize the asymptotic symmetry group BMS3 of three-dimensional flat space in terms of vertex operators on the string worldsheet. This provides an embedding of the BMS3 algebra in a consistent theory of quantum gravity.

Dmitry Tonkonog (Cambridge): *Elliptic relation for Floer homology*

Let f,g be two commuting symplectomorphisms of a symplectic manifold. One can define an action of f on the Floer homology HF(g) and similarly an action of g on HF(f). I will prove that the supertraces of these two actions are equal. This was proposed by Paul Seidel who suggests to call it the elliptic relation. The proof uses a gluing argument from Symplectic Field Theory. I will apply the elliptic relation to compute Floer homology of Dehn twists in

projective hypersurfaces and show they have infinite order in the symplectic mapping class group.