Acklam's Approximate Normal Quantile in Mathematica

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Acklam's Normal Quantile Function

Here it is in *Mathematica,* based on the web page at:

http://home.online.no/~pjacklam/notes/invnorm/

I have copied the data electronically from this page and the results are as Acklam claims.

```
AcklamQuantile[u_] :=
 Module[
  {a = Reverse[{-39.69683028665376, 220.9460984245205, -275.9285104469687,
      138.3577518672690, -30.66479806614716, 2.506628277459239}],
b = Reverse[{-54.47609879822406, 161.5858368580409, -155.6989798598866,
      66.80131188771972, -13.28068155288572}],
   c = Reverse[{-0.007784894002430293, -0.3223964580411365, -2.400758277161838,
      -2.549732539343734, 4.374664141464968, 2.938163982698783}],
   d = Reverse[{0.007784695709041462, 0.3224671290700398,
      2.445134137142996, 3.754408661907416}]},
  Which[0.02435 ≤ u ≤ 0.97575,
   Module[{v = u - 1 / 2, r = (u - 1 / 2) ^ 2},
    v * (a[[1]] + r * (a[[2]] + r * (a[[3]] + r * (a[[4]] + r * (a[[5]] + r * a[[6]]))))) /
      (1 + r * (b[[1]] + r * (b[[2]] + r * (b[[3]] + r * (b[[4]] + r * b[[5]])))))],
   u > 0.97575,
   Module[{q = Sqrt[-2 * Log[1 - u]]},
    - (c[[1]] + q * (c[[2]] + q * (c[[3]] + q * (c[[4]] + q * (c[[5]] + q * c[[6]]))))) /
     (1 + q * (d[[1]] + q * (d[[2]] + q * (d[[3]] + q * d[[4]]))))],
   True,
   Module[{q = Sqrt[-2 * Log[u]]},
     (c[[1]] + q * (c[[2]] + q * (c[[3]] + q * (c[[4]] + q * (c[[5]] + q * c[[6]]))))) /
     (1 + q * (d[[1]] + q * (d[[2]] + q * (d[[3]] + q * d[[4]]))))]]]
```

Evaluation

```
AcklamQuantile[0.975]
```

1.95996

Here is the Quantile based on *Mathematica*'s built in InverseErf:

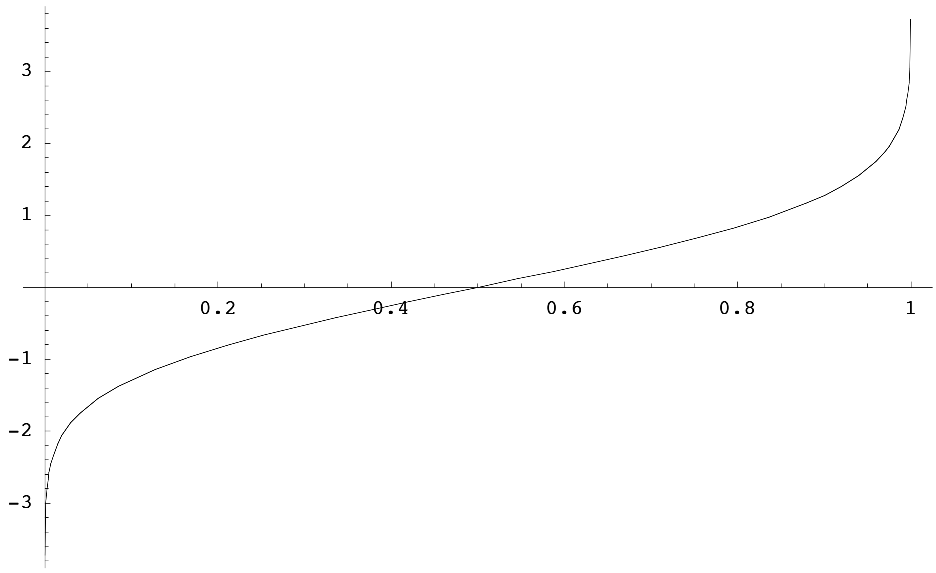
```
QuantileN[u_] := Sqrt[2] InverseErf[2 u - 1]
```

```
QuantileN[0.975]
```

1.95996

Plot

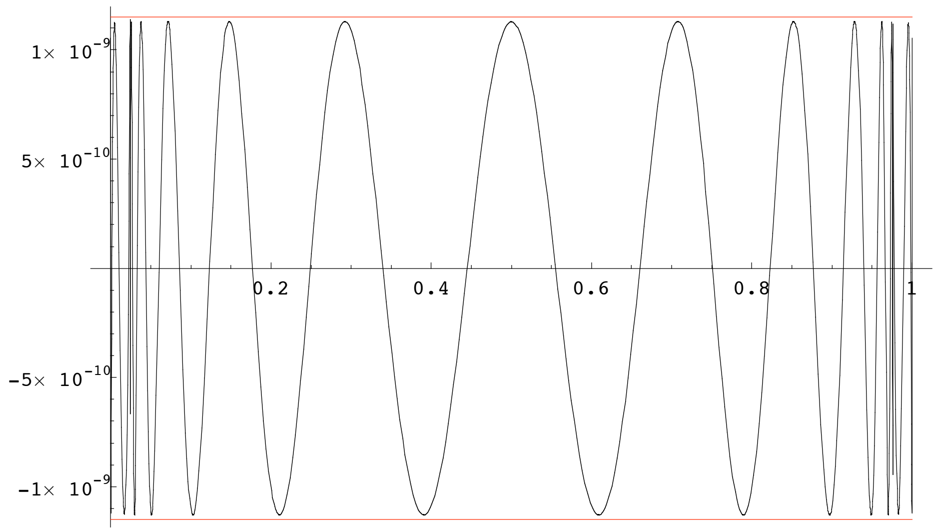
```
Plot[AcklamQuantile[u], {u, 0.0001, 0.9999}];
```



Relative Error against Mathematic Erf-based Quantile

So we just test the claim that the relative error does not exceed 1.15 E-9. A further probe into the tails confirm all is OK to at least $u = 10^{-22}$.

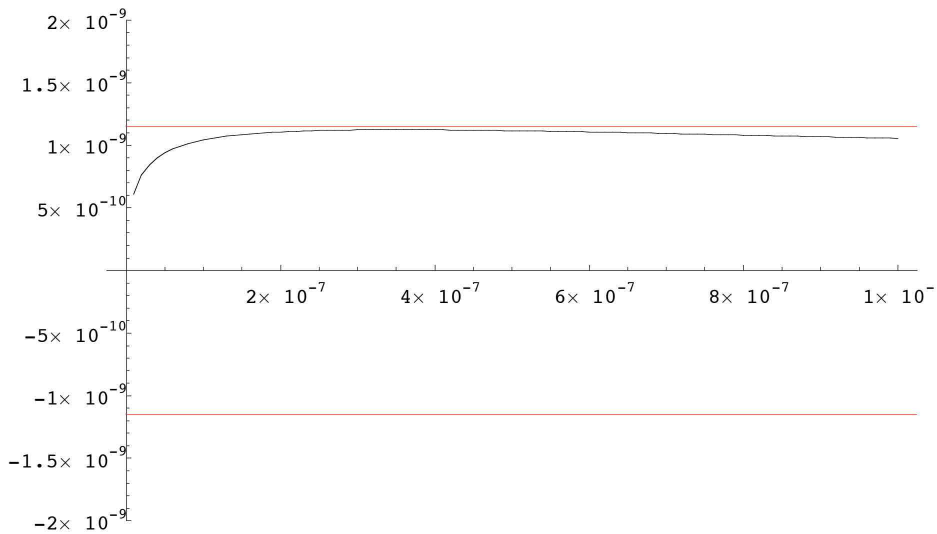
```
Plot[AcklamQuantile[u] / QuantileN[u] - 1, {u, 0.000001, 0.999999}, PlotPoints → 300,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, -1.15 10^-9}, {1, -1.15 10^-9}}]}];
```



Tail analysis:

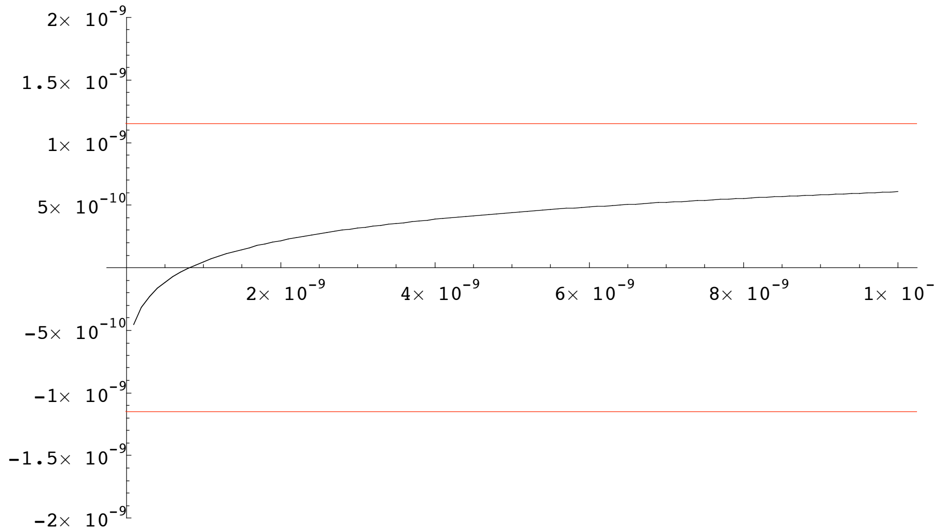
```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-8), 10^(-6), 10^(-8)}];

ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```



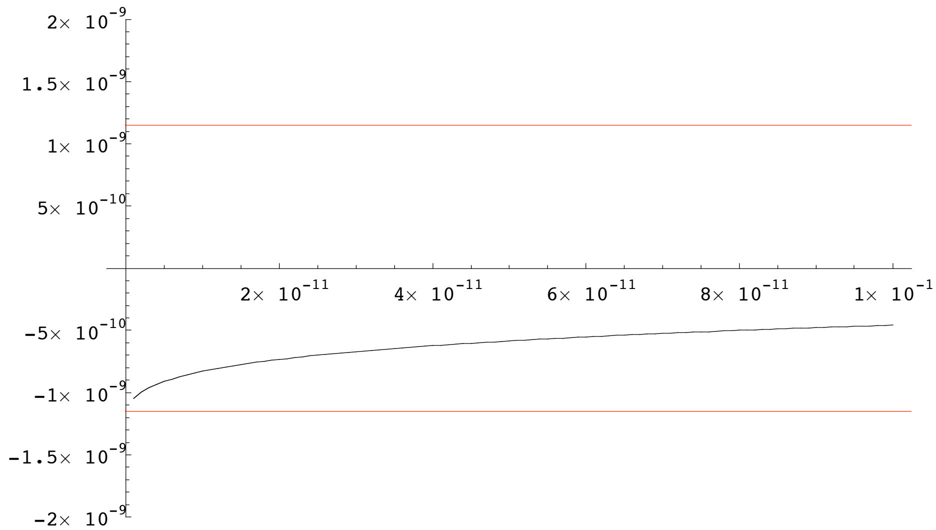

```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-10), 10^(-8), 10^(-10)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```



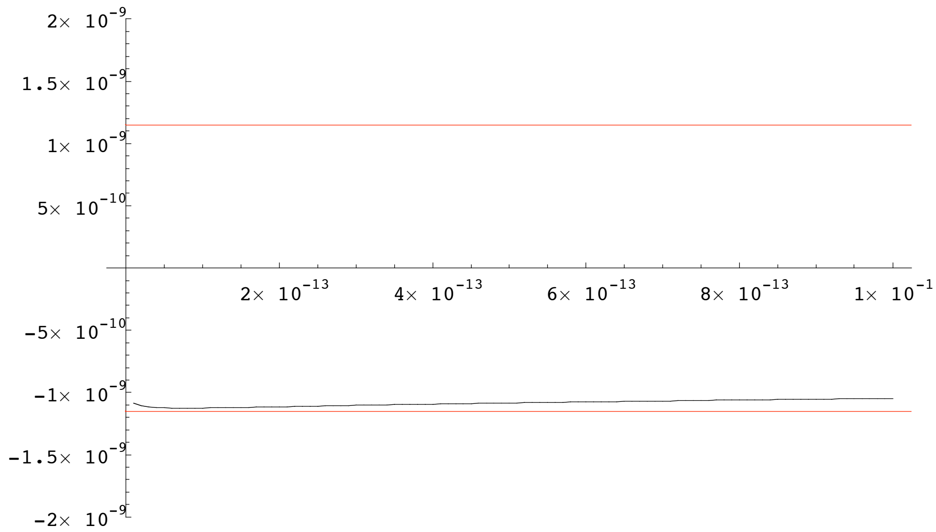

```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-12), 10^(-10), 10^(-12)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```



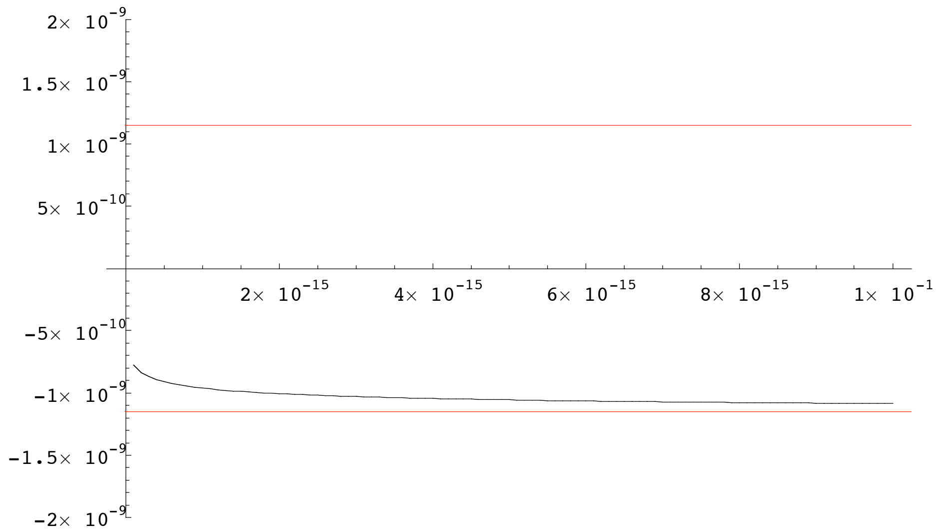

```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-14), 10^(-12), 10^(-14)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```



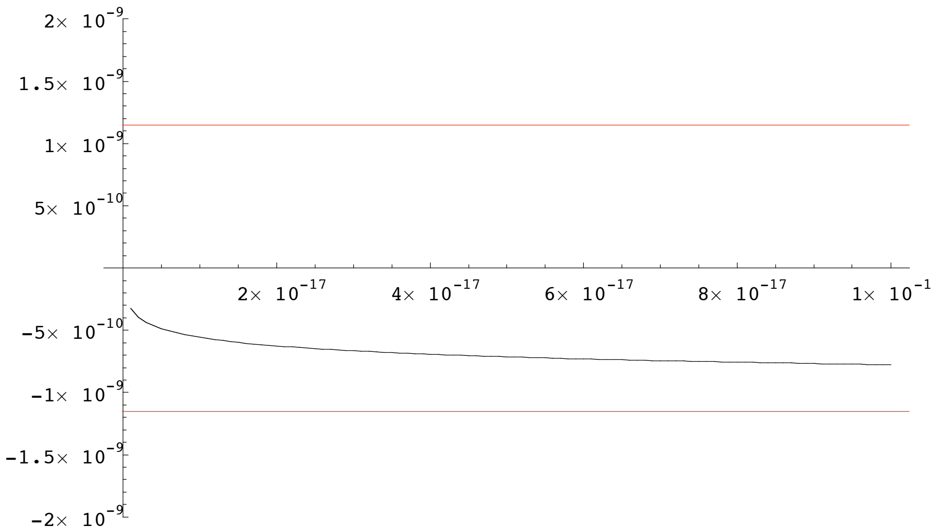

```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-16), 10^(-14), 10^(-16)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```



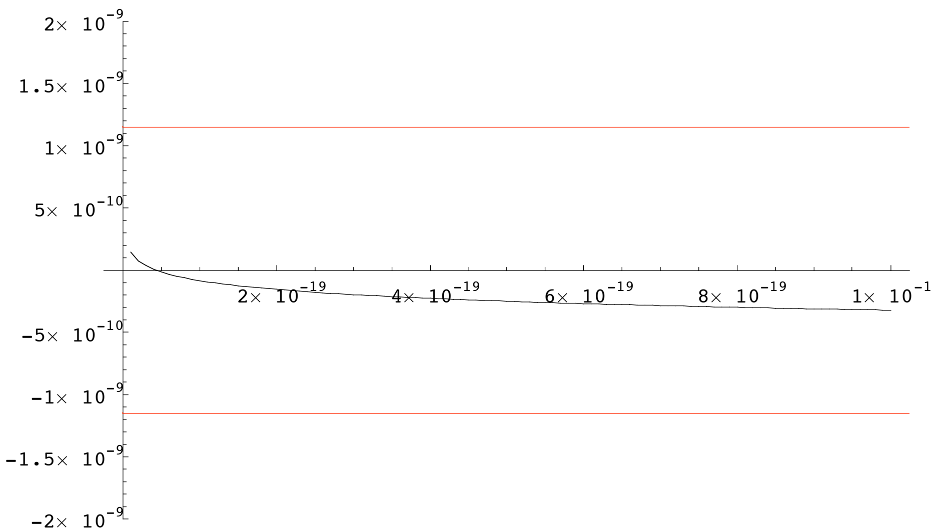

```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-18), 10^(-16), 10^(-18)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```




```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-20), 10^(-18), 10^(-20)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```




```
edata = Table[{u, AcklamQuantile[u] / N[QuantileN[u], 200] - 1},
   {u, 10^(-22), 10^(-20), 10^(-22)}];
```

```
ListPlot[edata, PlotJoined → True,
  Epilog → {RGBColor[1, 0, 0], Line[{{0, 1.15 10^-9}, {1, 1.15 10^-9}}],
    Line[{{0, - 1.15 10^-9}, {1, -1.15 10^-9}}]},
  PlotRange → {-2 10^(-9), 2 10^-9}];
```