Scattering Theory

Tentative syllabus for a postgraduate course. Based on the framework of five 2 hour lectures.

Lecture 1: Revision of spectral theory for self-adjoint operators. Classification of spectrum: point spectrum, essential spectrum, absolutely continuous and singular continuous spectrum. Spectral multiplicity. Weyl's Theory on the invariance of the essential spectrum under the compact perturbations. Discussion: Scattering theory as perturbation theory of continuous spectrum. Under what conditions on self-adjoint operators A and B, the a.c. parts of A and B are unitarily equivalent? We seek intertwining operators for A and B, how to construct them? Overview: wave operators; abstract approach, applications to the Schrödinger operator; scattering matrix; one dimensional vs. multi-dimensional problems.

Lecture 2: Wave operators: definitions, existence, completeness, isometry. The intertwining property. Existence of $W_\pm(B, A)$ is equivalent to completeness of $W_\pm(A, B)$. Two methods of establishing the existence and completeness: the trace class scheme and the smooth scheme. A sample result in the trace class scheme: Kato-Rosenblum theorem (without proof). A sample result in the smooth scheme: Friedrichs model, perturbation has a Holder continuous kernel (no proof). Exercise: Prove that the wave operators cannot exist as norm limits unless $A = B$.

Lecture 3: Discussion: Scattering theory as a comparison of two dynamics. (Example: Scattering theory in classical mechanics). The scattering operator and scattering matrix in quantum scattering. General operator theoretic definition, unitarity. Example: Schrödinger operator. Existence of wave operators by Cooke's method in quantum scattering; application to the Schrödinger operator.

Lecture 4: Stationary approach to scattering theory. Reduction of the existence of wave operators to the limiting absorption principle (LAP). Proof of the LAP in the trace class scheme. Proof of the LAP in the smooth scheme (for strongly smooth perturbations).

Lecture 5: Stationary representation for the scattering matrix. Born approximation. Discussion: high energy asymptotics of the scattering matrix for the Schrodinger operator in the leading term is determined by the Born approximation. Sketch of proof of the asymptotics of the eigenvalues of the scattering matrix for the Schrodinger operator with potential of a power decay.