Program of a Mini-Course

"Resonances and Threshold Singularities for Magnetic Quantum Hamiltonians"

Lecturer: Georgi D. Raikov (PUC, Santiago de Chile)

1. Basic facts from the spectral theory of magnetic quantum Hamiltonians (Schrödinger, Pauli, and Dirac operators with magnetic fields): self-adjointness, gauge invariance, diamagnetic inequality, Aharonov-Casher theorem [1, 7]. Constant magnetic fields [1].

2. Berezin-Toeplitz operators and pseudodifferential operators with contravariant symbols [2, 20, 11, 14]. Asymptotic distribution of the discrete spectrum of 2D magnetic Schrödinger and Pauli operators [15, 16, 18, 9].

3. The Lifshits-Krein spectral shift function (SSF): revision of basic facts [3, 22]. The Gesztesy-Makarov-Pushnitski representation of the SSF [12, 13, 10]. Specific properties of the SSF for magnetic quantum Hamiltonians [6]. Threshold singularities of the SSF for 3D magnetic Schrödinger operators, generalized Levinson formulae [8]. Extensions to Pauli and Dirac operators [17, 21].

4. Resonances for the 3D Schrödinger operator with constant magnetic field and short-range electric potential: definition, resonance-free domains, existence of infinitely many resonances near each Landau level [4]. Main asymptotic term of the resonance counting function near a given Landau level [5]. Extensions to Pauli and Dirac operators [19].

Remark: The program is meant for four lectures of approximately 90 minutes. If necessary, the program could be shortened or extended.

References

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