

# Statistical Mechanics of Glassy and Complex Systems

## King's College London, May 26

### Abstracts

#### **Roberto C. Alamino and David Saad (Aston)** Properties of Sparse Random Matrices over Finite Fields

The work focuses on key typical properties of sparse random matrices with entries in  $\text{GF}(q)$ , the Galois field of prime order  $q$ . The properties were analysed in the thermodynamic limit of large matrices. Generalising the usual transformation from  $\text{GF}(2)$  to  $\{\pm 1\}$  based on the group homeomorphism between  $\text{GF}(q)$  under addition mod  $q$  and the complex  $q$ -th roots of unity, we map the problem into a system of interacting spins, allowing for the use of methods of disordered spin lattices in statistical physics. A replica symmetric approach is then used to calculate the average dimension of the null space, the eigenspaces and the eigenvalue distribution, showing that the average dimension of the null space is equal to  $1-M/N$ , where  $M \times N$  is the matrix dimensionality, in all cases studied. Using general arguments based on the analogy with thermodynamic quantities and the gauge invariance of the Hamiltonian, we show the absence of replica symmetry breaking. We also introduce new techniques for taking averages over matrices with random row/column connectivity, which are transparent and easy to implement. Using the same techniques, we also calculate the total and average number of matrices for given distributions. Our results have practical relevance in several areas which can be mapped onto arbitrary sparse graphs and hypergraphs.

#### **Alessia Annibale (KCL)** Quantifying and comparing complexity of cellular networks: structure beyond degree statistics

We show that for any observed cellular network there exists a well-defined canonical random graph ensemble that produces graphs with structural characteristics identical to (and controlled solely by) the degree statistics and the degree correlations of the given network. We construct this ensemble and study its mathematical properties. We then demonstrate how the ensemble generates various powerful quantitative tools for analysing cellular signalling networks at a macroscopic level (dependent on structural measures only, not on network size), such as (i) precise measures of network complexity, (ii) precise measure for quantifying structural distances between networks, and (iii) numerical algorithms for generating canonical 'null models' with prescribed macroscopic structural properties.

#### **Douglas Ashton (Bath)** Phase behaviour of highly size-asymmetric fluid mixtures

#### **Johannes van Baardewijk (KCL)** Quantum Statistical Physics of Glasses at Low Temperatures

We use imaginary time path-integral techniques to analyse the quantum statistical physics of a microscopic model of structural glasses at low temperatures. The model can be thought of as arising from a random Born von Karman expansion of the full interaction potential. Both perturbative and non-perturbative methods are used to evaluate the path integral. The model is shown to reproduce one of the salient glassy low temperature anomalies, viz. a nearly linear specific heat at low  $T$ .

#### **Gil Benkő (Imperial College)** Record dynamics in synchronization transients

Record dynamics with log-Poisson statistics for the record event times have been reported in a number of stochastic glassy dynamical systems. Here we show the occurrence of glassy and record dynamics in the transient of a fully deterministic dynamical system. We studied the stepwise establishment of synchronization in the system of dynamically coupled maps introduced by Ito & Kaneko (*Phys. Rev. Lett.*, 88, 2001 & *Phys. Rev. E*, 67, 2003), where the plasticity of dynamical couplings might be relevant in the context of neuroscience. Synchronization transients are affected by numerics, as unstable near-synchronization might be stable in limited precision calculations. An implementation of a higher precision numerical method was used that helps differentiating between unstable near-synchronization and stable synchronization.

#### **Martin Evans (Edinburgh)** Dynamics of Genetic Switches

We study the statistical properties of a simple genetic regulatory network. This network consists of a binary genetic switch in which stochastic flipping between the two switch states is mediated by a linear feedback mechanism. We present a general solution for the steady-state statistics in the on and off states, and for the flip time (persistence) distributions.

#### **Christopher Fullerton (Manchester)** The Growing Correlation Length in Glassy Systems

An important feature of glassy systems is the presence of a correlation length that grows as the temperature is lowered or the density is increased towards the jamming density. Supercooled liquids and binary systems are often used as model glass formers. Several different length scales can be defined for these systems and here three of them are outlined and their density dependence calculated in one-dimensional monatomic and binary systems. The relevance of these lengthscales to the behaviour of similar systems in higher dimensions is considered.

**Yan Fyodorov (Nottingham)** Prefreezing of multifractal exponents in Random Energy Models with logarithmically correlated potential

**Tobias Galla (Manchester)** (tbc) Intrinsic fluctuations in stochastic delay systems: theoretical description and application to a simple model of gene regulation

The effects of intrinsic noise on stochastic delay systems is studied within an expansion in the inverse system size. We show that the stochastic nature of the underlying dynamics may induce oscillatory behaviour in parameter ranges where the deterministic system does not sustain cycles, and compute the power spectra of these stochastic oscillations analytically, in good agreement with simulations. The theory is developed in the context of a simple one-dimensional toy model, but is applicable more generally. Gene regulatory systems in particular often contain only a small number of molecules, leading to significant fluctuations in mRNA and protein concentrations. As an application we therefore study a minimalistic model of the expression levels of *hes1* mRNA and Hes1 protein, representing the simple motif of an auto-inhibitory feedback loop and motivated by its relevance to somite segmentation.

Reference: <http://arxiv.org/abs/0901.3271>

**Juan P Garrahan (Nottingham)** Large deviation methods and the dynamics of glasses, *Science* 323, 1309 (2009)

A natural statistical mechanics formalism to study systems with complex dynamics is the large deviation formalism. Large deviation functions are to dynamics what free energies are to thermodynamics. In this talk I will describe how to apply this formalism to the glass transition problem. I will show using large deviation function methods that simple models of glasses, the so-called kinetically constrained models, display a first-order phase transition in the space of trajectories. A manifestation is the phenomenon of dynamic heterogeneity. I will also describe how to calculate large deviation functions in atomistic models, via numerical techniques such as transition path sampling, and discuss the implications of this space-time transition scenario to the glass transition problem in general.

**Harry Goldingay & Jort van Mourik (Aston)** Agent based optimization in a threshold model for distributed task assignment

We propose multi-agent threshold based decentralized algorithm to maximize efficiency and flexibility in a model for distributed task assignment. Using a genetic algorithm, we provide new rules that outperform existing ones both in static and dynamic environments, and observe extinction and speciation. From a more theoretical point of view, we analyze finite size effects and causes of efficiency loss, we derive **exact dynamics** and theoretical upper bounds on the efficiency for the memoryless algorithm. Finally, we show that the inclusion of memory leads to emergent cooperation between agents and a significant increase in efficiency, without influencing the excellent scalability.

**Eytan Katzav (KCL):** Packing of elastic rods – a Statistical Physics Approach

We present a statistical approach for studying the close packing of elastic rods in two and three dimensional containers. Using the free volume approach, we show that a self-reorganization of the rod above a critical density leads to a more efficient packing (“the matchbox effect”). In addition, we predict the existence of a jamming transition for high densities, hinting at the glassy character of this system. Finally, we discuss the relevance of the present results to physical and biological systems, such as DNA packaging in viral capsids, polymeric glasses and crumpled paper, in view of recent experiments and simulations. The advantage of this approach is that it puts on equal footing geometrical and mechanical aspects of the system.

**Renaud Lambiotte (Imperial College)** Laplacian Dynamics and Multiscale Modular Structure in Networks

Most methods proposed to uncover communities in complex networks rely on their structural properties. Here we introduce the stability of a network partition, a measure of its quality defined in terms of the statistical properties of a dynamical process taking place on the graph. The time-scale of the process

acts as an intrinsic parameter that uncovers community structures at different resolutions. The stability extends and unifies standard notions for community detection: modularity and spectral partitioning can be seen as limiting cases of our dynamic measure. Similarly, recently proposed multi-resolution methods correspond to linearisations of the stability at short times. The connection between community detection and Laplacian dynamics enables us to establish dynamically motivated stability measures linked to distinct null models. We apply our method to find multi-scale partitions for different networks and show that the stability can be computed efficiently for large networks with extended versions of current algorithms.

**David Lancaster (Westminster)** A Field Theoretic Approach to Euclidean Matching

We present, but fail to successfully analyse, a replica field theory describing the geometric combinatorial optimisation problem of matching.

**Juan Pablo Neirotti (Aston)** Adaptability and over-specificity in learning agents

In this talk I will show you the effects observed in the evolutionary process of learning agents when their fitness depends on both the difficulty of the environment and the cost of resources used. I will also present some analytical results linked to the asymptotic behaviour of the learning curve when the student uses an appropriate learning algorithm, in the on-line learning scenario.

**Tim Rogers (KCL):** Eigenvalue Distributions of Sparse Random Matrices

In 1976, Edwards and Jones demonstrated a bridge between statistical mechanics and the spectral density of real symmetric random matrices. In the case of sparse matrices, the resulting statistical mechanics problem had resisted exact solution for many years. In this talk I briefly outline some recent successes in the analysis of this problem, as well as an extension to non-Hermitian matrices.

**Nigel Wilding (Bath)** Computer simulations of polydisperse crystals