On pp 517–8 of the November 2006 Mathematical Gazette there is a "Feedback" from Michael de Villiers in which he states a theorem about a hexagon with opposite pairs of sides parallel, and about the lines joining the mid-points of opposite sides. The theorem stated is that these three lines are concurrent. This is not quite right, because if the conic is either a parabola or a parallel line-pair, then these three lines will be parallel; however, in the context of projective geometry this is just a matter of allowing the lines to be concurrent at a point at infinity.

For the proof, he correctly asserts that the hexagon must be inscribed in a conic, but he then claims this conic can be projected to a circle so that the opposite sides, in the image, are still parallel, and the mid-points of sides are still midpoints. This can only be done if the conic is an ellipse, and then the projective transformation required is actually an affine transformation (which means it will preserve parallels, and mid-points). But if the conic is a hyperbola or a parabola, it won't work, because although the conic can be projected to a circle, parallels and mid-points will not be preserved. And if the conic is improper (a line-pair) then it cannot be projected to a circle at all.

A correct proof, for a proper conic, is via harmonic properties. Let $ABCDEF$ be a hexagon (in the projective plane), with opposite sides AB, DE; BC, EF; CD, FA meeting at P, Q, R respectively. If P, Q, R are collinear then (as we know) by the converse of Pascal's theorem, $ABCDEF$ lie on a conic S, which we now suppose to be proper. Let G, H, I, J, K, L be the harmonic conjugates of P, Q, R, P, Q, R w.r.t. A, B; B, C; C, D; D, E; E, F; F, A respectively. Then GJ, HK, IL are the polars of P, Q, R respectively, w.r.t. S. It follows that they are concurrent, at the pole M of the line PQR. If we now move to the Euclidean interpretation where the opposite sides of $ABCDEF$ are parallel, then PQR is the line at infinity, so its pole M (if finite) is the centre of S; and the harmonic conjugates G, H, I, J, K, L are the midpoints of the various sides of the hexagon. But if S is a parabola, then M is at infinity—this is the case where PQR touches S, at M—so in the Euclidean interpretation, the lines GJ, HK, IL are then parallel, rather than concurrent. (They are parallel to the axis of S.)

In the other case, where S is a line-pair, it is not necessary to use projective geometry, and it is an easy exercise for the reader to show that the join of the mid-points of two parallel chords is either concurrent with the two lines of the line-pair (if they meet), or else is parallel to them (if they do not meet).