DEAR EDITOR,

Bill Richardson solves the old chestnut "How many presents did my true love send to me?" in the November 2002 *Gazette* (p. 468) by summing the triangular numbers 1, 3, 6, ... corresponding to each day's haul. There is a direct route to his answer: notate each present by an integer triple (r, s, t), where this stands for the rth present of type s received on day t. So, for example, $(3, 5, 8)$ stands for the 3rd of the 5 gold rings received on day 8.

We now have to count the triples (r, s, t) with $1 \leq r \leq s \leq t \leq 12$, or (equivalently), if we put $R = r$, $S = s + 1$ and $T = t + 2$, we must count the triples (R, S, T) with $1 \leq R < S < T \leq 14$, and plainly the answer is the number of ways of choosing 3 objects from 14, or $\binom{14}{3}$.

For n days, just replace 12 by n, and the answer is $\binom{n+2}{3}$. These numbers are sometimes called *tetrahedral* numbers, and it should be clear that if we want to sum the first n tetrahedral numbers, a similar technique involving quadruples of integers will yield the result. Indeed, triangular numbers can be produced this way also: for

$$1 + 2 + 3 + \ldots + n = 1 + (1 + 1) + (1 + 1 + 1) + \ldots,$$

and if we let (r, s) stand for the rth 1 in the sth bracket, then we are counting solutions of $1 \leq r \leq s \leq n$, or $1 \leq R < S \leq n + 1$, which gives the number of ways of choosing 2 objects from $n + 1$, or $\binom{n+1}{2}$.

Yours sincerely,

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