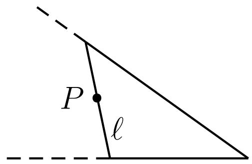
1. For given perimeter, what shape rectangle has the greatest area? (Proof by geometry!)

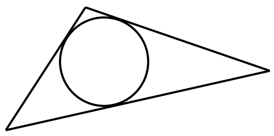
2. (Token "maths in context" question.) Farmer George has a certain length[1] of (flexible) fencing which is to be used to enclose the greatest possible rectangular area of a large[2] (plane[3], level) field, using a (straight!) field boundary as one side of the rectangle. What shape rectangle should he—oops, sorry—*she* use? [[1]Yes, I know that if I want to be taken seriously by educationalists I should say 289·6 metres, or something equally ridiculous, but I have better things to do with my time. [2]That is, *large enough* so that the other field boundaries don't get in the way. [3]You may ignore the curvature of the earth.]

3. If all four side-lengths of a quadrilateral are fixed (but the angles are variable), what shape should it be to have greatest area?

4. A variable line ℓ through a fixed point P meets, on either side of P, two fixed lines. How should ℓ be placed to make the triangle so formed of least area?

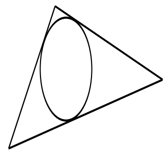


5. Describe how to position three tangents to a fixed circle so as to enclose it in a triangle of least area.

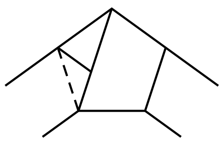


6. For given perimeter, what shape triangle has the greatest area?

7. Given a triangle, which ellipse, situated within the triangle, has greatest area?

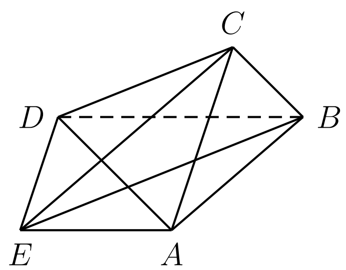


8. Tie the strip of paper you have been given into a common or garden *overhand* or *thumb* knot, and gently pull it tight, flattening as you go. Prove that the shape you get is a regular pentagon.

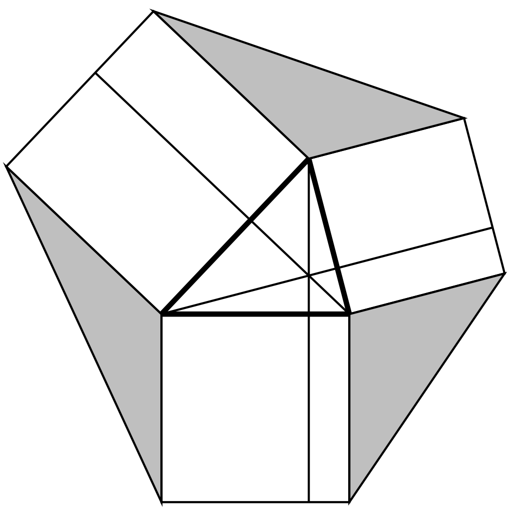


9. Let $ABCDE$ be a convex pentagon, with $AB \parallel EC$, $BC \parallel AD$, $CD \parallel BE$, and $DE \parallel CA$. Prove that $EA \parallel DB$, and that

$$\frac{EC}{AB} = \frac{AD}{BC} = \frac{BE}{CD} = \frac{CA}{DE} = \frac{DB}{EA} = \frac{1+\sqrt{5}}{2}.$$



10. Given an acute-angled triangle (shown bold, in the diagram), erect a square on each side, drop perpendiculars, and join up, as shown. The perpendiculars divide the three squares into six rectangles. Show that for each vertex of the triangle, the two rectangles that meet there have the same area. Show also that each of the shaded triangles has the same area as the original triangle.



What modifications are needed if the original triangle is obtuse-angled?

JRS