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Coursework Solution and Revision Lecture

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Lecture Plan

- Coursework feedback - good for preparing for dissertation and exam.
- PTES (Postgraduate Taught Experience Survey)
- Some questions from Solutions to Mock Exam.

Coursework

I used the following basic mark scheme, but there was some leeway, for example, if your answer was particular well-written you might have received some extra credit for that.

- 15% Basic MATLAB - for example does the code run
- 15% Basic Presentation - is it adequately presented in LaTeX
- 30% Mathematical description of what you have done
- 20% Accuracy of answers
- 20% Testing

Summary of what was needed

- How you simulated the stock price.
- How you chose the price of the derivative.
- How you computed the delta of the derivative.
- How you simulated delta hedging.
- How you tested your results.

Which do you think was the hardest part?

Testing

- Testing is the hardest part. So let's think about testing from the beginning.
- Were you 99% sure your answer was correct?
- Were your tests completely automated?
- How could I be completely confident to mark 90 different scripts with 90 different questions. Is it possible that I made a mistake?
- To get 100% you needed to be as confident as me.

Testing strategy

- Divide the code into lots of functions and test each part separately.
  - To test the calculation of $\overline{\sigma}$ you could ...
  - To test the pricing of the derivatives you could ...
  - To test the Euler scheme simulation of the stock price you could ...
  - To test the simulation of delta hedging you could ...
  - The main complexity of this question on top of what was done in lectures was ...
- How could you test this aspect of your answer specifically?
- Which aspect of the testing do you think was worth the most marks?

My solution 1

Throughout my solution, I wrote all my code so that it could price options and simulate hedging for any stock price model of the form

$$\mathrm{d}S_t = S_t(\mu \mathrm{d}t + \sigma(t)\mathrm{d}W_t).$$

with σ a function. The standard Black-Scholes model and the model given in the question are special cases. This allowed me to use the standard Black-Scholes model as a test case.

- What kind of option were you asked to price?
- How were you asked to price the derivatives?
- How did you calculate the root mean squared volatility? Did you discuss this?
- How much credit would you get for pricing the derivative by Monte Carlo?
- How should you compute the delta of the derivative? Did you discuss this?

My solution 2

I used MATLAB's `integral` function to compute the root mean squared value of $\sigma(t)$ over a given time interval $[t, T]$ given by

$$\bar{\sigma}_t = \left(\int_t^T \frac{\sigma(\tau)^2}{T-t} \mathrm{d}\tau \right)^{\frac{1}{2}}.$$

I tested this by comparing the results with the analytic formula for the integral when $\sigma(t) = 1 + t + t^2$.

Student answer

One student wrote a very similar answer to this part when I looked at their code, but if I looked at their description what they said they had done was

I wrote two functions

- RMSVolatility, which computes the root mean squared volatility $\overline{\sigma}_t$ for a given t, and
- digitalCallPrice which computes the digital call option price.

By not telling me that they had written their code so it worked with any $\sigma(t)$ they missed some marks as writing the code to be more general is a key part of the testing strategy in my view.

Another student wrote

$$\sigma = \left(\int_t^T \frac{\sigma(\tau)^2}{T-t} \mathrm{d}\tau \right)^2$$

Some other student answers

- Some students computed the root mean square error by Simpson's rule. This was wasted effort as there is no point doing this in practice. The same goes for the Rectangle rule. No marks lost.
- Some students computed the root mean square error by Monte Carlo. This is a silly way to compute the integral. Perhaps one mark lost.
- Some students computed the integral analytically. This is fine. But you need to tell me that you did this. I don't think any student used the word analytically. You should then tell my how you tested your analytic integration, presumably you would do this numerically instead.

To test σ and Δ, I use the integrand from problem to calculate σ and Δ when $t=0$,and compare this result to my function.

My solution 3

Following the guidance in the question I computed the price of option C_t and the delta Δ_t by modifying the standard formulae for a digital call option [?]. Thus

$$C = e^{-r(T-t)} N(d_2) \tag{1}$$

$$\Delta_t = e^{-r(T-t)} \frac{N'(d_2)}{\overline{\sigma}_t S \sqrt{T-t}}. \tag{2}$$

where

$$d_2 = \frac{\log(S/K) + (r + \frac{1}{2}\overline{\sigma}^2)(T-t)}{\overline{\sigma}_t \sqrt{T-t}}$$

and N is the c.d.f. of the normal distribution.

Remarks

- Pricing the wrong kind of option was perhaps the most common error.
- Some of you had to price ordinary puts and calls, some had to price digital options.
- "I didn't know the formula for a digital call option" Then look it up or compute it yourself. You had several weeks to complete this coursework so there is no excuse for not finding something you can Google in minutes.
- If you really can't cope with a digital option. At least state very clearly "I couldn't answer the question as set with a digital option, so I have answered the question for a vanilla option with payoff instead."
- The payoff at the end of the delta hedging process will also be affected by the hedging function given. Many of you simply copied the call payoff from lectures blindly.

My solution 4

I tested my computation of C and Δ_t in the case of constant σ against the values produced by an online calculator. I also priced the option using the Monte Carlo method and checked that the results were the same within a 95% confidence interval. To do this I simulated the stock in the $\mathbb{Q}$-measure and then computed the discounted expected value. I discuss how the stock price was simulated in the $\mathbb{Q}$-measure below.

To test my computation of Δ_t, I compared (2) to the numerical approximation:

$$\Delta = \frac{\partial C}{\partial S} \approx \frac{C(S+h) - C(S-h)}{2h}$$

where $C(S)$ is given by (1) thought of as a function of S.
Many students write $=$ when they should write $\approx$.

Simulating the stock price

- You were asked to simulate the stock price to test the effectiveness of delta hedging. Should the simulation be in the $\mathbb{P}$ or $\mathbb{Q}$ measure?
- What does it actually mean to say you should simulate the stock price using the Euler scheme? Did you do this?

My solution 4

I divided $[0, T]$ into N equal intervals of length $\delta t \frac{T}{N}$. I simulated in the $\mathbb{P}$-measure using the Euler scheme, this gave the difference equation:

$$S_{(i+1)(\delta t)} = S_{i\delta t} + S_{i\delta t}\left(\mu\,\delta t + \sigma(t)\sqrt{\delta t}\epsilon_i\right)$$

where the ϵ_i were simulated independent normally distributed random variables of mean 0 and standard deviation 1.

Popular errors included using one of the following symbols: $\overline{\sigma}_t$, σ, $\mathrm{d}t$, r. It was also very common to use the Euler scheme for the log of the stock price, which is not what was asked.

My solution 5

I tested my simulation of S_t by computing the mean and standard deviation of $s_T := \log S_T$ in the case of constant σ and small δt. According to general theory this should be close to mean and s.d. of the true distribution of s_T which we now compute. By Ito's lemma:

$$\mathrm{d}(s_t) = (\mu - \frac{1}{2}\sigma^2)\mathrm{d}t + \sigma \mathrm{d}W_t$$

so $s_T \sim \mathcal{N}(\log(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T)$.

By performing the simulation with $\mu = r$, I simulated stock prices in the risk-neutral measure as required to price the option by Monte Carlo. The consistency between the analytic pricing formulae and my answers gave me confidence in my answer in the case when σ was not constant.

Simulating delta hedging

- How did delta hedging for this question differ from the delta hedging code used in the course?
- Did you clearly emphasize that difference in your mathematics.

My solution 6

I simulated delta hedging by using the following formulae to compute the bank balance at each time, $b_{i\delta t}$ for $i \in \{0, 1, 2, \ldots N\}$. At time 0 the trader charges C and purchases Δ_0 units of stock, hence

$$b_0 = C - \Delta_0 S_0$$

At time $i\delta t$ the trader receives interest and purchases $\Delta_{i\delta t} - \Delta_{(i-1)\delta t}$ units of stock so their bank balance becomes

$$b_{i\delta t} = e^{-r\delta t} b_{(i-1)\delta t} - (\Delta_{i\delta t} - \Delta_{(i-1)\delta t}) S_{i\delta t}.$$

At the final time T the trader receives interest, liquidates their stock portfolio and pays the payoff to their customer. Hence

$$b_T = e^{-r\delta t} b_{(N-1)\delta t} + \Delta_{(N-1)\delta t} S_T - \text{payoff}(S_T).$$

Note that at each time Δ_t was computed using the formula (2).

Student answers

- Many students blindly copied the payoff $(S-K)^+$ from the lectures even if it wasn't relevant.
- It should be totally unambiguous how Δ_t was computed.

My solution 7

To test the simulation of delta hedging code, I checked that

$$B_T := \sqrt{E_{\mathbb{P}}(b_T^2)}$$

approached zero as $\delta \to 0$. I confirmed that this was the case by plotting a log-log plot of B_T againt δt shown in figure .... To turn this into an automated test I asserted that B_T was less than ... when $\delta t = 0.001$.

Note that the correct place for a chart is in your write-up, not in an automated test.

My solution 8

Note that throughout this project, I seeded the random number generator at the beginning of all tests to ensure that the tests were reliable even when they depended upon the generation of random numbers.

My solution 9

The numerical results I obtained were

Price	...
Mean final bank balance	...
s.d. of final bank balance	...

As a final, simple sanity check I noted that the mean final bank balance was close to zero relative to the standard deviation of the final bank balance. I also compared the price to the price given for the derivative by an online calculator when I used the value $\sigma = \ldots$. (If pricing a digital, mention that the price was between 0 and 1 as expected).

Weak descriptions

Here were some proposed tests

- To test integrateSigmaFunction: Given t = 0, it is easy to calculate whether the optput is 0:0574.
- Calculate the payoff when stock price is sufficiently larger than strike price.

What is wrong with what has been written?

One common error

- Suppose that $\mu = r$ and you charge the correct price but don't hedge. What will be the mean bank balance at the end?
- What does checking that the mean bank balance is approximately zero really check?

Learning from experience

- Writing clear mathematics is hard. But if an examiner can't understand what you wrote they won't give you any marks.
- You will not get credit in your project for what you thought, only for what you actually wrote.
- The examiner of your project will look at what you wrote and only glance at your code. So if your code is interesting, say what is interesting.
- Nobody is impressed by irrelevant information. You make it easier for yourself and for the examiner if you focus on stating what is important clearly.
- My general guidance would be to ask: what is the examiner going to think is important? Focus on that. For example, I think it was fairly obvious that I was interested in testing so that is an area I think you should have focussed on too.

More professional LaTeX

The following looks awful in my view

$$Final balance = e^{-r(T-t)} balance + delta S_T - payoff$$

Use `\text` and `\times` to improve this

$$\text{Final balance} = e^{-r(T-t)}\text{balance} + \text{delta} \times S_T - \text{payoff}.$$

The LaTeXcode is `\text{delta} \times S_T`. You should use `\log` and `\sin` for $\log$ and $\sin$ too.

First impressions count. If your dissertation looks good then you will get more marks.

Exercise (10 mins)

- I've written up my answer as a LaTeX document `https://nms.kcl.ac.uk/john.armstrong/courses/fm06/revisionlecture2018/modelanswer.pdf`. Compare it to your answer.
  - Did I emphasize different points from you? Why do you think I did that?
  - To what extent did I discuss the details of the MATLAB code?
  - Do you think what you wrote feels as "professional" as my answer? If not what could you do to give a more professional answer?
  - Did you proof-read your work carefully before handing it in?
- Was there anything I mentioned that you did too but that you didn't actually mention in your write up?

PTES

- Help us improve our programme!