King's College London

University Of London

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

ATTACH this paper to your script **USING THE STRING PROVIDED**

Candidate No: ..... **Desk No:** .....

MSc Examination

7CCMFM06 Numerical and Computational Methods in Finance

Mock Exam

Time Allowed: Two Hours

All questions carry equal marks. Full marks will be awarded for complete answers to FOUR questions. Only the best FOUR questions will count towards grades A or B, but credit will be given for all work done for lower grades.

NO calculators are permitted.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. (i) State the rectangle rule for integrating a real valued function $f : [a, b] \longrightarrow$ R defined on a closed interval. [**20%**]

(ii) Write the MATLAB code to integrate e^{-x^2} over the interval $[0, 1]$ using the rectangle rule. [**30%**]

(iii) Name three other numerical integration techniques that you could use to evaluate this integral and sketch a log-log plot of their convergence as the number of steps increases. Your plot should indicate how rounding errors on a digital computer limit the maximum accuracy. [**30%**]

(iv) Explain how pricing options by Monte Carlo simulation can be interpreted in terms of numerical integration. [**20%**]

2. (i) Write a MATLAB function to simulate stock prices price paths that follow the Black–Scholes model with given parameters. [**30%**]

(ii) Suppose that a trader writes a call option at the Black–Scholes price and then performs discrete time delta hedging up to the maturity of the option. They rebalance their portfolio at time points $\{0, \delta t, 2\delta t, 3\delta t, \ldots T\}$. Any money not invested in the stock is invested in a bank account which grows at the risk free rate r.

(a) Write down difference equations for the number of assets held at each time point. [**40%**]

(b) Sketch a histogram of the expected profit and loss of this hedging strategy [**10%**]

(iii) Explain briefly what is meant by gamma hedging and explain why a trader might choose to gamma hedge an exotic option. [**20%**]

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3. (i) Draw a table summarizing the numerical methods for risk neutral option pricing that were taught in this course and indicate which of these methods can be used to price the following types of option:

(a) A European call option

(b) An American put option

(c) An Asian call option

(d) An up-and-out option.

[**30%**]

(ii) When pricing a European put option by the finite difference method, what boundary conditions would you use? [**20%**]

(iii) Recall that the Black–Scholes PDE is:

$$V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S - rV = 0$$

where subscripts denote partial differentiation. Use this to derive the difference equations that must be solved to price a put option by the explicit finite difference method [**30%**]

(iv) How do the difference equations change when pricing an American put option? [**20%**]

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4. (i) What is meant by a pseudo square root of a positive definite symmetric matrix A? [**10%**]

(ii) What is meant by the Cholesky decomposition of a positive definite symmetric matrix A? [**10%**]

(iii) Compute the Cholesky decomposition of the matrix:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

[**30%**]

(iv) Explain how you would generate a sample of random variables X_1 and X_2 from a two dimensional multivariate normal distribution with mean 0 and covariance matrix:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

[**20%**]

(v) Describe how you can simulate stock prices in discrete time that approximately follow the Heston model:

$$dS_t = \mu S_t\, dt + \sqrt{\nu_t} S_t\, dW_t^S$$

$$d\nu_t = \kappa(\theta - \nu_t)\, dt + \xi\sqrt{\nu_t}\, dW_t^\nu$$

where W_t^S and W_t^ν are Wiener processes with correlation ρ, S_t is the stock price at time t, ν_t is the volatility process and all other terms are constants.

5. (i) What is meant by VaR and CVaR? [**20%**]

(ii) What is the sub-additivity property of a coherent risk measure? Show that VaR is not sub-additive. [**20%**]

(iii) Write a difference equation you could use to simulate a stock price that follows the Black-Scholes model. [**20%**]

(iv) Describe how you could use the results of such a simulation to estimate the VaR of a call option on a stock. [**20%**]

(v) Explain briefly how you could go about testing the results of this calculation. [**20%**]

Final Page