Worksheet 8

Many of these questions require you to think about the boundary conditions. To choose the boundary conditions we must estimate the value of the derivative on the boundary. Often we will not know the exact value on the boundary.

Example: We argue that on the top boundary for a call option, the chance of the stock hitting the top boundary, $S_{\max}$ and yet ending out of the money is negligible (so long as we choose $S_{\max}$ to be sufficiently large). Hence on the top boundary, we can replicate the option by a holding of one unit of stock and -K zero coupon bonds. So the value on the top boundary is approximately $S_{\max} - e^{-r(T-t)}K$.

It is important to choose a value of $S_{\max}$ to ensure that falling from $S_{\max}$ to below K over a time period less than T is very unlikely. We know that the change in $\log S$ over time T is normally distributed with mean $(r - \frac{1}{2}\sigma^2)T$ and standard deviation $\sigma\sqrt{T}$. A move of 4 standard deviations is unlikely, so we could take $S_{\max} = e^{-(r-\frac{1}{2}\sigma^2)T+4\sigma\sqrt{T}}K$.

1) [⋆] When valuing a put option using the Black–Scholes PDE method, what would you choose as boundary conditions. Be sure to specify what value of $S_{\max}$ you would use. Value a put option using the Black–Scholes PDE method. Test your answer. (You may choose the market data, strike etc. yourself).

(Solution: see the file `testPricePutByExplicitMethod.m` in `lecture8.zip`)

2) [⋆] When valuing a put option using the heat equation method, what would you choose as boundary conditions? Value a put option using the heat equation method. (You may choose the market data, strike etc. yourself).

(Solution: see the file `testPricePutByHeatEquationExplicit.m` in `lecture8.zip`)

3) [⋆] To value an up and out knock out call option with barrier B, what would you choose as your boundary condition? Use the explicit finite difference method to price a knock out call option. Test your answer. (You may choose the market data, strike etc. yourself).

(Solution: see the file `testPriceKnockOutByExplicitMethod.m` in `lecture8.zip`)

4) [⋆] Use the explicit finite difference method to price a knock-in option. Test your answer.

(Solution: see the file `testPriceKnockInByExplicitMethod.m` in `lecture8.zip`)

5) Use Matlab's `surf` or `mesh` commands to reproduce the 3-d plot given in the lectures.

(Solution: see the file `generateSurfacePlot.m` in `lecture8.zip`)

6) [⋆] Price an American put option, choose the market data etc. yourself. You might want to test your answer using the Bloomberg terminals.

(Solution: see the file `testPriceAmericanPutByExplicitMethod.m` in `lecture8.zip`)

7) [⋆⋆] May 2017, Q5

8) [⋆⋆] May 2014, Q5.