King's College London

University Of London

PLACE this paper and any answer booklets **in the EXAM ENVELOPE provided**

Candidate No: .... **Desk No:** ....

MSc Examination

7CCMFM06 Numerical and Computational Methods in Finance

May 2018

Time Allowed: Two Hours

All questions carry equal marks.
Full marks will be awarded for complete answers to all four questions.

You are permitted to use a Calculator.
Only calculators from the Casio FX83 and FX85 range are allowed.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. (i) Describe mathematically how you could simulate M stock price paths in the Black-Scholes model at the discrete time points $(0, \delta t, 2\delta t, \ldots, N\delta t = T)$. Justify your answer [**30%**].

(ii) A trader sells a call option with strike K and maturity T for the Black-Scholes price P. They then delta hedge the option at the discrete time points defined above. Derive finite difference equations for the trader's bank balance b_i at each time point $i\,\delta t$. [**30%**]

(iii) A second trader chooses not to trade in options at all. They have an initial principal P which they initially invest entirely in stock. At each subsequent time point they:

- invest all their wealth in the stock if the stock price increased over the last time interval
- otherwise they place all their wealth in a risk-free bank account.

Let b_i denote their bank balance, q_i denote the quantity of stock they hold and W_o denote their total wealth (all taken at time $i\,\delta t$). Derive finite difference equations for these quantities which allow their wealth at each time to be computed. [**20%**]

(iv) In the Black-Scholes model, how would you expect the expected return of each trader to depend upon the drift of the stock μ assuming that the time interval δt is small? Justify your answer. [**20%**]

2. (i) Write a MATLAB function that approximates the integral

$$\int_{-1}^{1} e^{\cos(x)} \mathrm{d}x \tag{1}$$

using the Monte Carlo method. [**30%**]

(ii) If X and Y are random variables with finite mean and variance, compute the variance of $X + \lambda Y$, where λ is a real number, in terms of the variance and covariance of X and Y. [**10%**]

(iii) For what value of λ is this covariance minimized? [**10%**]

(iv) Use your answer to the questions above to describe the *control-variate method* for improving the accuracy of Monte Carlo integration. [**10%**]

(v) Suggest an appropriate control variate to improve the calculation of the integral (1). Justify your answer. [**20%**]

(vi) Could antithetic sampling be used to improve the calculation of the integral (1). Justify your answer. [**10%**]

(vii) How would you test your MATLAB function? (You must not use the MATLAB function `integral` in your answer to this part.) [**10%**]

3. (i) Let $f(x)$ be a smooth function. What is meant by the forward, backward and central estimates for the derivative? [**20%**]

(ii) Write a MATLAB function to compute the central estimate for the derivative of a function f. [**20%**]

(iii) Write a unit test for this function. You may use the function `assertApproxEqual` that was defined in the lectures if you wish. [**20%**]

(iv) Write the MATLAB code to generate a log-log plot illustrating the error in the estimate. You should use built-in function `loglog(x,y)` to draw a log-log plot of the points x against the points y. [**20%**]

(v) What would you expect the plot to look like? Justify your answer. [**20%**]

4. (i) What is meant by Cholesky decomposition? [**20%**]

(ii) Explain how Cholesky decomposition can be used to simulate a multivariate normal distribution with mean vector μ and covariance matrix Σ. [**20%**]

(iii) Suppose that a financial market consists of n assets whose value at time T follow such a multivariate normal distribution. Suppose also the initial prices of the assets are given by the components of a vector c. An investor has an amount P_1 to invest at time 0 and they wish to purchase a portfolio of assets with an expected payout of P_2. They wish to choose a portfolio that minimizes the risk of their position. Short selling is allowed. Describe mathematically what optimization problem they should solve, being careful to justify how you measure risk. [**20%**]

(iv) How could you solve this optimization problem in MATLAB? [**10%**]

(v) In practice, the covariance matrix Σ and mean μ would need to be estimated from historic data. Describe briefly how you might use computer simulations to estimate the magnitude of this model risk. [**30%**]