King's College London

University Of London

PLACE THIS PAPER AND ANY ANSWER BOOKLETS **in the EXAM ENVELOPE provided**

Candidate No: .... **Desk No:** ....

MSc Examination

7CCMFM06 Numerical and Computational Methods in Finance

Summer 2016

Time Allowed: Two Hours

All questions carry equal marks.
Full marks will be awarded for complete answers to FOUR questions.
Only the best FOUR questions will count towards grades A or B, but credit will be given for all work done for lower grades.

NO calculators are permitted.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. (a) Write a Matlab function to compute

$$\int_0^1 \frac{1}{\sqrt{1+\sin^2(t)}}\,\mathrm{d}t \qquad (1)$$

using the Monte Carlo method with N samples. Your function should compute both the integral and an estimate of the error. [**40%**]

(b) Describe a better numerical method to compute the integral given in equation (1). Justify your answer. [**20%**]

(c) Name a technique you can use to improve the accuracy of the Monte Carlo methods and describe briefly how you would apply it to this problem. [**20%**]

(d) What one dimensional integral do you need to compute the price of a call option in the Black–Scholes model? [**20%**]

2. (a) A stock price S_t follows the stochastic process given by:

$$\mathrm{d}S_t = S_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_t)$$

where μ and σ are constants and W_t is a Wiener process. Find a function $f(S, t)$ such that $f(S_t, t)$ follows a Brownian motion with drift 0 and volatility σ. [**20%**]

(b) The Black–Scholes PDE is:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

What change of variables would you use to transform this to the heat equation. Justify your answer. [**20%**]

(c) Suppose that you wish to price the following options using the explicit finite difference method, which equation would you solve and what would be the boundary conditions? Justify your answers.

(i) A European put option. [**30%**]

(ii) An up and out knockout call option. [**20%**]

(d) What are the pros and cons of the implicit and explicit methods of solving partial differential equations by finite differences? [**10%**]

3. (a) The stochastic differential equation (SDE) for geometric Brownian motion is:

$$\mathrm{d}S_t = S_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_t)$$

(i) Write down the difference equation for the Euler scheme for this stochastic differential equation. [**10%**]

(ii) Describe a better method to simulate S_t. Explain your answer. [**10%**]

(iii) Write the MATLAB code to simulate S_t. [**20%**]

(b) A trader believes that the Black–Scholes model holds. She writes a European call option at the Black–Scholes price with strike K and maturity T and delta hedges her position at the times $(0, \delta t, 2\delta t, 3\delta t, \ldots)$.

(a) Derive difference equations for the value in her risk free account at times $i\delta t$ [**30%**]

(b) Sketch a graph showing how you would expect her profit and loss to be distributed if she is correct. How will your graph change as δt is reduced? [**10%**]

(c) Suppose that in fact there is a 1% bid-ask spread at all times that she has forgotten to take into account. How would the graphs change? Explain your answer. [**20%**]

4. (a) Define the term *pseudo square root.* [**10%**]

(b) Define the term *Cholesky decomposition.* [**10%**]

(c) Explain why Cholesky decomposition is useful for simulating stochastic processes. Give a financial example of when you might use it. [**30%**]

(d) Find the Cholesky decomposition of the following matrix

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

where $(-1 < \rho < 1)$. [**20%**]

(e) Find two more pseudo square roots of this matrix. [**30%**]

5. (a) Write pseudo code to show how you would compute the price of an up and out call option with strike K and barrier B by the Monte Carlo method in the Black–Scholes model. [**40%**]

(b) Describe how you could compute the delta of the option by the Monte Carlo method. [**20%**]

(c) How would the accuracy of your computation of the delta be related to the size of the Monte Carlo simulation? Give the mathematical reason for your answer. [**10%**]

(d) Describe how you would test your computation of the delta. [**30%**]