King's College London

UNIVERSITY OF LONDON

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Candidate No: .... **Desk No:** ....

MSC EXAMINATION

7CCMFM06 NUMERICAL AND COMPUTATIONAL METHODS IN FINANCE

SUMMER 2013

TIME ALLOWED: TWO HOURS

ALL QUESTIONS CARRY EQUAL MARKS. FULL MARKS WILL BE AWARDED FOR COMPLETE ANSWERS TO FOUR QUESTIONS. ONLY THE BEST FOUR QUESTIONS WILL COUNT TOWARDS GRADES A OR B, BUT CREDIT WILL BE GIVEN FOR ALL WORK DONE FOR LOWER GRADES.

WITHIN A GIVEN QUESTION, THE RELATIVE WEIGHTS OF THE DIFFERENT PARTS ARE INDICATED BY A PERCENTAGE FIGURE.

NO CALCULATORS ARE PERMITTED.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. **Monte Carlo.** Consider a stock which follows a geometric Brownian motion $dS_t = rS_t dt + \sigma S_t dW_t$ (under the risk neutral measure). Assume that the values for the risk-free rate r and the volatility σ are known. W_t is a standard Wiener process.

(a) Explain how you would apply Ito's formula to produce the stochastic differential equation for the logarithm of the price $X_t = \log S_t$. Explain how you would simulate N random paths of X_t over a discrete set of points $(0, \Delta t, 2\Delta t, \ldots, M\Delta t = T)$. Why would one prefer to simulate the logarithm X_t rather than the underlying price S_t directly? [**40%**]

(b) Consider a European call option with exercise price K and maturity T. Use the paths that you generated in (a) to estimate the price of this option. Produce an estimate of the simulation error involved. How would your code change if there was a barrier, say $B > S_0$, which would cancel the contract if the price S_t were to hit this barrier (from below) before expiry. [**30%**]

(c) Produce pseudo-code that produces prices for the call option and the corresponding barrier option, using the methodology that you described in part (b). Give a short overview of the 'antithetic variables technique', and show how your code should be altered to implement this technique. [**30%**]

(End of Question 1)

2. Hedging.

(a) Suppose that we have positions on two derivatives which have the same underlying asset. Show that if we merge these positions then the Greeks (Delta, Gamma, Vega) of the composite portfolio will be equal to the sum of the corresponding Greeks of the individual assets. Use this principle to build the appropriate strategies that would Delta, Delta-Gamma and Delta-Gamma-Vega hedge a given derivative. [**50%**]

(b) A portfolio manager is holding a derivatives portfolio with exposure to the FTSE index, and is willing to Delta-Gamma-Vega hedge his exposure. What instruments would you recommend as appropriate hedges? What practical considerations would you highlight? [**20%**]

(c) Assume that you are given a portfolio of derivatives and you have access to the pricing function $F(S,t)$ and also functions that produce the portfolio Delta (say $\Delta(S,t)$) and Gamma (say $\Gamma(S,t)$) for any combination of the spot price S and time t. Give the principles and high level pseudo-code that would hedge this exposure, following your answers in (a) and (b). [**30%**]

(End of Question 2)

3. Volatility.

(a) Describe the ‘implied volatility surface’ and the stylised patterns it exhibits for equity index derivatives. Discuss not only the static patterns but also its dynamic behaviour. How do changes of the volatility surface relate to changes of the underlying index? [**40%**]

(b) Explain the main differences between a ’stochastic volatility’ and a ’local volatility model’. Give the corresponding stochastic differential equations to highlight the differences and explain their implications. [**30%**]

(c) Say that we have a set of T historical returns, say $\{R_t\}_{t=1}^{t=T}$ at our disposal. Show how the Exponentially Weighted Moving Average scheme can be produced to filter out historical volatility. Give the relevant pseudo code. [**30%**]

(End of Question 3)

4. Finite differences. The Black-Scholes PDE is $f_t + (r - \sigma^2/2)f_s + \frac{1}{2}\sigma^2 f_{ss} = rf$, for a pricing function $f = f(s,t)$, where s is the log-price of the underlying and t denotes the time. The interest rate r and volatility σ are assumed known. Subscripts denote partial derivatives.

(a) Write down the finite difference approximations for the first and second order derivatives involved, and derive the implicit scheme. Assume that the interest rate is zero. You are pricing a bet that the underlying asset price, S_t, will remain between B_L and B_H up to time T, with $B_L < S_0 < B_H$: the payoff of the contract is zero if the underlying asset hits either of the barriers between times zero and T; the payoff is one at time T if the price of the underlying remained between the barriers at all times. Describe the appropriate boundary conditions and show how the PDE is solved numerically. **[50%]**

(b) Produce pseudo-code that implements your answer to (a) and produces the price of this bet at time $t = 0$. Show graphically how you would expect the pricing function to look like at time $t = 0$. **[30%]**

(c) How would you produce estimates for the Delta and Gamma (at time $t = 0$) using the implementation that you described in (a)? How would you alter the pseudo-code in (b) to also produce estimates for the Delta and Gamma? How would you expect the Delta and Gamma to look like with respect to the underlying price? How would you produce an estimate of the Vega for this bet (at time $t = 0$)? **[20%]**

(End of Question 4)

5. Risk/SDEs.

(a) Explain the concept of Value-at-risk (VaR). Explain why VaR is a superior measure of risk when compared to the volatility as a measure of risk. [**40%**]

(b) Consider the stochastic integral $J_t = \int_0^t B_u dB_u$, where B_u is a standard Brownian motion. Produce pseudo-code that will simulate paths of the standard Brownian motion B_u and also the corresponding trajectory of the integral J_t. [**30%**]

(c) Follows (b). Using the appropriate change of variable apply Ito's formula and solve the stochastic integral J_t. Produce the pseudo-code that will simulate the explicit solution for J_t. Why would the paths produced by simulating this explicit solution be more accurate than the paths for J_t that you produced in (b)? [**30%**]

(End of Question 5)