King's College London

UNIVERSITY OF LONDON

ATTACH THIS PAPER TO YOUR SCRIPT **USING THE STRING PROVIDED**

Candidate No: .... **Desk No:** ....

MSC EXAMINATION

7CCMFM06 NUMERICAL AND COMPUTATIONAL METHODS IN FINANCE

SUMMER 2020

TIME ALLOWED: TWO HOURS

ALL QUESTIONS CARRY EQUAL MARKS. FULL MARKS WILL BE AWARDED FOR COMPLETE ANSWERS TO FOUR QUESTIONS. ONLY THE BEST FOUR QUESTIONS WILL COUNT TOWARDS GRADES A OR B, BUT CREDIT WILL BE GIVEN FOR ALL WORK DONE FOR LOWER GRADES.

NO CALCULATORS ARE PERMITTED.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. The price of a stock S_t at time t follows the Black–Scholes process

$$\mathrm{d}S_t = S_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t)$$

with S_0 given.

Let $(0, \delta t, 2\delta t, \ldots, N\delta t = T)$ a sequence of evenly placed time points. An discrete time knock out option pays out 0 if the stock price is above a barrier B at any of these time points. Otherwise, it pays out $\max\{S_T - K, 0\}$ at maturity.

(a) How could you simulate stock prices at the given time points? Explain how your answer follows from the Black Scholes model. [**40%**]

(b) What process would you need to simulate in order to price the option by Monte Carlo? Describe how you could use the results of the simulation to approximate the risk neutral price of the option. [**20%**]

(c) How would you estimate the error of the option price? [**10%**]

(d) Sketch a graph showing how you expect the price of option to vary as the barrier changes. [**20%**]

(e) Suppose you have written a function that can price such a knockout option given the various market and contract parameters. How could you test this function? [**10%**]

2. (a) Define Value at Risk (VaR) and Expected shortfall. Describe one advantage and one disadvantage of each of these approaches to measuring risk. [**20%**]

(b) Describe the three main (VaR) computation methodologies: parametric VaR, Monte Carlo VaR and historic VaR. Summarize the key advantages and disadvantages of each approach to computing VaR. [**60%**]

(c) Write a MATLAB function to compute the 99% historical 1 day VaR of a position in a single stock given: a vector of consecutive end of day stock prices; the quantity of the stock held; the current price of the stock. For simplicity assume that interest rates are zero and that there are no gaps between days in the vector of stock prices. [**20%**]

3. (a) Describe the delta hedge trading strategy. [**10%**]

(b) Suppose a trader writes a European call option for a price P and performs delta hedging at fixed time points $(0, \delta t, 2\delta t, \ldots N\delta t = T)$. Assume money not invested in stock grows at a risk free rate r. What would be the cash flows and trades at each time point? [**20%**]

(c) Suppose we have already written a MATLAB function `generatePrice-Paths` to simulate stock prices and a function `blackScholesDelta` to compute the Delta of an option, write pseudo-code that simulates the strategy. [**30%**]

(d) Suppose that the `generatePricePaths` function uses the BlackScholesModel, and suppose that the price P is the BlackScholesPrice, sketch a histogram of the final profit made by the trader. [**10%**]

(e) Sketch a log-log plot to show the rate of convergence as N is increased. Clearly state what you are plotting on each axis and how the plot should be interpreted. [**20%**]

(f) Add to your log-log plot a line indicating what happens if we include transaction costs in the simulation. [**10%**]

4. (a) State the rectangle rule for integrating a real valued function $f : [a, b] \to \mathrm{R}$ defined on a closed interval. [**10%**]

(b) Write a MATLAB function to implement the rectangle rule showing how it can be used to integrate e^{-x^2} over the interval $[a, b]$. [**40%**]

(c) Give an example of a transformation you could apply to convert an integral over an infinite interval to an integral over a finite interval. There is no need to consider issues of convergence. [**10%**]

(d) Sketch a log-log plot showing the convergence of the following three integration algorithms as the number of steps increases.

(i) The trapezium rule

(ii) Simpson's rule

(iii) Monte Carlo integration

Your plot should not show the theoretical result, instead it should show the actual results that would be achieved on a digital computer. Describe how your plot can be interpreted. [**20%**]

(e) Explain what is meant by a pricing kernel and how one can be used to price options by numeric integration. [**20%**]

5. (a) Complete a table showing which techniques, as taught in the course, can be used to price the given types of options on a single stock.

	Rectangle rule	Monte Carlo	Finite Difference
European Call Option			
Knockout Call Option			
Asian Call Option			
American Call Option			

[**20%**]

(b) When the risk free interest rate is zero, the Black–Scholes partial differential equation simplifies to:

$$C_t + \frac{1}{2}\sigma^2 S^2 C_{SS} = 0$$

Show that this can be transformed by a change of variables into a constant coefficient equation:

$$C_t = -\frac{1}{2}\sigma^2 C_{xx}$$

[**10%**]

(c) What is the explicit finite difference scheme for solving this equation? Describe how you could use this scheme to numerically price an option on a stock obeying the Black–Scholes model.

[**30%**]

(d) Explain how the explicit finite difference scheme can be interpreted in terms of a trinomial tree. [**20%**]

(e) What is meant by numerical stability? Under what condition is this scheme stable? [**20%**]