James Eells

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Jim Eells was a mathematician of wide interests and considerable influence. He moved to Britain from the United States in the late 60's and has had a major, and continuing, effect on the development of mathematics here. His appreciation of the difficulties faced by scientists, and mathematicians in particular, working in third world countries led to his becoming the first Director of a new mathematics section at the International Centre for Theoretical Physics at Trieste: one of the Centre's main aims is to give third world scientists the opportunity to keep up to date with rapidly moving areas of science and mathematics. He was a man of irrepressible enthusiasm for mathematics, and most other things; especially people, irreverent fun, lots of wine, and music. His enthusiasm for the latter ranged across most styles, and included delighting the younger children of his colleagues with lively performances of scatological songs.

Born in 1926 in Cleveland, Ohio, he went to school at Western Reserve Academy until his exuberance led to his expulsion. However he was admitted to Bowdoin College, a distinguished liberal arts college in Maine, with the idea of studying English Literature, Mathematics, and much more. He graduated in 1947. The College's avowed aims for its students include:

> "To be at home in all lands and all ages;
> To count Nature a familiar acquaintance,
> And Art an intimate friend; To gain a standard for the appreciation of others work
> And the criticism of your own;
> To carry the keys of the worlds library in your pocket,
> And feel its resources behind you in whatever task you undertake;
> To make hosts of friends...
> Who are to be leaders in all walks of life;
> To lose yourself in generous enthusiasms
> And cooperate with others for common ends "

They could hardly have been more successful in Jim Eells' case; (and recognised this later by the award of an honorary Doctorate of Laws in 1996).

After Bowdoin he had a "gap" year teaching mathematics at Robert College (now Boğaziçi University) Istanbul. He returned to the USA, to be an Instructor in mathematics at Amherst, during which time he met his future wife Nan. After two years he decided that his future really was to

be in mathematics and enrolled for a PhD at Harvard under the topologist and analyst Hassler Whitney. In 1954 he, with Nan, two young daughters and PhD, set off on his academic career. This started at the Institute for Advanced Study at Princeton, went on to the University of California at Berkeley, returned to the East Coast for a position at Columbia University, New York; and in 1964 after a year at Churchill College, Cambridge, he took a full professorship at Cornell University.

He returned to Cambridge for 1966-67; two events that year gave a foretaste of the future. In December he visited Ghana, in connection with Churchill College, to give a mathematics course, having persuaded a Cornell colleague, Cliff Earle, to make a follow up visit early in 1967. Between the two visits they began fruitful joint research, and Eells was invited to run a small symposium in the summer of 1967 in Warwick with Earle as one of the principal participants. The innovative mathematics department of the new University of Warwick had already gained an international reputation and in 1969, excited by the freedoms and potential in Warwick he joined the mathematics department there, becoming its first Professor of Analysis. This was with the strong support of his family, now including four children.

It was an appointment which fitted perfectly with the philosophy of the department at that time, which was to feature research in global, rather than traditional, analysis; and it was already getting a reputation as a centre for the global approach to dynamical systems theory. It is tempting to describe global analysis as a holistic approach to mathematics. In it the whole geometry or topology of the spaces involved play a role rather than just the equations describing the behaviour or motion in small areas. Non-linearity, especially that caused by curvature is a prevalent aspect. A prime example is Eells' most famous article. Written with J. Sampson of John Hopkins University it founded the theory of "harmonic maps" and the "non-linear heat flow". The latter describes the behaviour of one shape stretched over another. For example think of an uninflated balloon wrapped around the surface of a sphere, anchor ring, or of some other shape, (even a Klein bottle), and allowed to settle down, relaxing, but only moving on the surface. Strange shaped balloons are also allowed, for example water rings, or just a piece of balloon with ends fixed. The final position of minimal stretching would depend on the shape of the balloon, the shape of the surface, and the way it was wrapped. For example you might force the balloon over the sphere before sealing the end, or you might just lie the balloon slightly stretched over part of the surface. The final result is the "harmonic map". If you did it with a rubber band rather than a balloon the harmonic maps would be along geodesics, lines of shortest length (possibly just one point). These generalisations of geodesics were found to have profound importance in various parts of geometry, and in some aspects of theoretical physics such as string theory. The "settling down" motion, governed by the non-linear heat equation, was essentially the first example of a "geometric evolution", the study of which has been of increasing importance and finally appears to have led to a proof of the famous "Poincaré conjecture ", one of the classical problems for which there is a million dollar prize. In fact in lectures and articles in the early '60's he gave the Poincaré conjecture as a potential application of the general approach he was following.

Infinite dimensional objects such as the space of all paths on a surface often occur in global analysis, and one of Eells' early works was to show that many such spaces have a "smooth" structure which enables calculus to be applied on them. He also played a leading role in the classification of such infinite dimensional manifolds. Global analysis and probability theory were combined in foundational work in the analysis of Brownian Motion on curved spaces, such as the surface of spheres. One result was that if you roll a surface along the trajectory of a typical Brownian

path on an ink covered plane the mark the ink makes on the surface will be the trajectory of a "Brownian particle on the surface". A difficulty here is that Brownian particles move so irregularly that standard, Newtonian, calculus cannot be applied so even what it means to "roll" is unclear. It has to be replaced by Itô calculus, the calculus which is also now in regular use in mathematical finance. From the mid '70's, however, the main part of his work was related to harmonic map theory.

The highly successful, year long, Warwick Symposia he organised, "Global Analysis" 1971-72 and "Geometry of the Laplace Operator" 1976-77, to a large extent introduced novel mathematics to Britain, and together with his many, and energetically pursued connections abroad, attracted many fine PhD students and research fellows from overseas (and the UK) to study with him. After the first of these, in the summer of 1972, he "took the Symposium" to the International Centre for Theoretical Physics at Trieste for the first session held there on mathematics.

The Trieste Centre had been founded in 1964 by Nobel Laureate Abdus Salam, with support from the Italian Government, UNESCO, and the Atomic Energy Authority, "to foster advanced studies and research, especially in developing countries". The latter was achieved mainly by bringing researchers from such countries to the Centre for research conferences, schools, and just periods of peace in which to study. Jim Eells' interest in helping develop the careers of younger mathematicians extended especially to those from the third world and his Ghana visit had made him appreciate their need. The session he ran at Trieste in 1972 was an early move in the development of a Mathematics Division at ICTP, of which he became the first Director from 1986 to 1992, on partial secondment from Warwick. His research activities in Warwick were barely diminished however, and if anything there were more visitors and visiting scholars, together with another year long Symposium, "Partial differential equations in differential geometry", in 1989-90.

While at Warwick he had already travelled widely and this was increased as part of the Director's role in Trieste. Jim Eells' mathematical influence internationally became even more substantial, both personally, as a mentor to those progressing through their career, and scientifically through the spread of his work and that which was built on it. He had at least 38 graduate students most of whom became academics. They have held positions in universities from New Zealand to California, via India, Iceland, Portugal, Belgium, Italy, Denmark, and France, and of course the United Kingdom. However there are many more throughout the world who owe their careers to him. Many, together with their families will also especially remember Nan Eells for her friendship and excellent dinner parties with only mild attempts to control the exuberance of her husband.

He retired from Warwick in 1992, moving to live in Cambridge. He continued working on harmonic maps and travelling abroad to discuss mathematics or for conferences. During this period he also helped to set up a regular UK- Japan Winter School in Mathematics. His final monograph, on harmonic maps between polyhedra, co-authored by the Danish mathematician Fuglede, was published in 2001. His mathematical legacy to Warwick still thrives there.

James Eells. mathematician; born Cleveland, Ohio, 25th October 1926 ; married Anna Munsell, 1950 {three daughters, one son}; died Cambridge 14th February 2007.