Keio University COE :
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Progress in Mathematics Motivated by Natural and Social Phenomena

Ireland–Japan Workshop 2006

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Organizer: Jurgen Berndt (University College Cork)
Martin Guest (Tokyo Metropolitan University)
Yoshiaki Maeda (Keio University)

December 18

13:00-14:00 Hiroshi Tamaru (Hiroshima University)
Cohomogeneity one actions on symmetric spaces of rank one, and of higher rank

14:30-15:30 Tatsuya Tate (Nagoya University)
A spectral analogue of asymptotics of the number of partitions

16:00-17:00 Michel Stolz (Rhur University)
On random elements of compact symmetric spaces

December 19

10:00-11:00 Alfonso Gracia-Saz (Keio University)
Symbols of functions of operators and higher-order Bohr-Sommerfeld rules

11:30-12:30 Ramiro Carrillo (Tokyo Metropolitan University)
Pfaffian systems from twistor fibrations

14:00-15:00 Martin Guest (Tokyo Metropolitan University)
Pfaffian systems from harmonic maps

15:30-16:30 Keizo Yamaguchi (Hokkaido University)
Parabolic Geometries associated with Differential Equations of Finite Type

17:00-18:00 Benjamin McKay (University College Cork)
Smooth projective planes and symplectic topology

December 20

10:00-11:00 Toru Morimoto (Nara Women University)
Graded Lie algebras, geometric structures and differential equations on filtered manifolds

11:30-12:30 Hajime Sato (Nagoya University)
Orbits of differential equations under actions of diffeomorphism groups

14:00-15:00 Toshikazu Miyashita (Komoro High School)
3-graded decompositions of exceptional Lie algebras $\g$ of type E_7 and group realizations of $\g_{ev}, \g_0$ and $\g_{ed}$

15:30-16:30 Dmitri Zaitsev (Trinity College)
Which manifolds bound Levi-flat hypersurfaces?

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Alfonso Gracia-Saz (Keio University)

"Symbols of functions of operators and higher-order Bohr-Sommerfeld rules"

Abstract:

In the quantum description of a physical system, the observables are represented by operators on a Hilbert space. In the classical description, they are represented by functions on a Poisson manifold. Weyl quantization provides a bijection between quantum and classical observables. To every (quantum) operator ¥hat{A}, we associate a (classical) function A , called its symbol. We consider the following problem. Let ¥hat{A} be an operator with symbol A and let f be a smooth function. Then ¥hat{B} := f(¥hat{A}) is another operator, with symbol B . What is B in terms of A? We will provide an answer to this question in the form of a formula "a la Feynman", i.e., a power series whose terms are labeled by diagrams.

As an application, we will discuss Bohr-Sommerfeld rules, an asymptotic method to obtain the eigenvalues of an operator.

Ben McKay (University of College Cork)

"Smooth projective planes and symplectic topology"

Abstract:

Using symplectic topology and the Radon transform, we prove that smooth 4-dimensional projective planes are diffeomorphic to the standard complex projective plane.

Dmitri Zaitsev (Trinity College)

"Which manifolds bound Levi-flat hypersurfaces?"

Abstract:

In this joint work with P. Dolbeault and G. Tomassini we study the problem of characterising real submanifolds in ${¥mathbb C}^n$ that bound Levi-flat hypersurfaces, the CR analogues of minimal surfaces. In constrast to the extensively studied case $n=2$, in higher dimenion $n>2$, a boundary of a Levi-flat hypersurface must satisfy a nontrivial local condition at its CR points, the so-called nowhere minimality. There is also a flatness condition at the complex points, where the complex tangent space dimension (the CR-dimension) ``jumps". We show that these

necessary conditions are essentially sufficient for the given submanifold to be a boundary of an ``immersed Levi-flat subvariety'', an immersed Levi-flat ``hypersurface'' with singularities. The proof ingredients include the Reeb-Thurston stability theorem as well as a parametric version of the Harwey-Lawson theorem on the existence of complex-analytic chains with prescribed boundaries.

Michel Stolz (Rhur University)

"On random elements of compact symmetric spaces"

Abstract:
In this talk I will present joint work with Benoit Collins (Lyon). We study random elements of Cartan-embedded compact symmetric spaces and prove asymptotic Gaussianity for various statistics of the matrix entries. The proofs are based on the invariant theory of the classical groups.

Martin Guest (Tokyo Metropolitan University)

"Pfaffian systems from harmonic maps"

Abstract:
The harmonic map equations can be regarded as a Pfaffian system, in general infinite-dimensional. We shall review this theory in the case of harmonic maps of finite uniton number, where the Pfaffian system is finite-dimensional, and discuss some open questions. This includes the case of harmonic maps arising from canonical twistor fibrations.

Ramiro Carrillo (Tokyo Metropolitan University)

"Pfaffian systems from twistor fibrations"

Abstract:
Canonical twistor fibrations lead to Pfaffian systems by means of their superhorizontal distibutions. We identify explicitly the Pfaffian systems of five or less variables that arise in this way. We also consider the infinitesimal symmetries of these Pfaffian systems.

Toshikazu Miyashita (Komoro High School)

"3-graded decompositions of exceptional Lie algebras $¥g$ of type E_7 and group realizations of $¥g_{ev}, ¥g_0$ and $¥g_ {ed}$"

Abstract:

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Toru Morimoto (Nara Women's University)

"Graded Lie algebras, geometric structures and differential equations on filtered manifolds"

Abstract:

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Hajime Sato (Nagoya University)

"Orbits of differential equations under actions of diffeomorphism groups"

Abstract:

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Tatsuya Tate (Nagoya University)

"A spectral analogue of asymptotics of the number of partitions"

Abstract:

The subject on asymptotics of the number of partitions for positive integers is one of main topics in classical analytic number theory. There are many results on this issue since the celebrated work by Hardy-Ramanujan appeard. Among these results, Meinardus gives a beautiful asymptotic formula for the number of partitions by using a variant of the saddle point method.

In this talk, we address a problem on asymptotics of the number of states of Boson gas, which is regarded as a spectral analogue of the number of partitions, whose Hamiltonian is given by a positive Elliptic pseudo-differential operator of order one on a compact manifold. We will give some background and motivation for the problem, and then give an asymptotic formula for the number of states of Boson gas. We use the Meinardus saddle point method combined with the Duistermaat-Guillemin trace formula for the wave operator to obtain the asymptotic formula.

Hiroshi Tamaru (Hiroshima University)

"Cohomogeneity one actions on symmetric spaces of rank one, and of higher rank"

Abstract:

Cohomogeneity of an isometric action on a Riemannian manifold is defined by the codimension of the regular orbit, therefore the cohomogeneity one action is the action with hypersurface orbit. I would like to talk about a sketch of classification of cohomogeneity one actions on noncompact symmetric spaces of rank one (especially on complex hyperbolic spaces), and some results for higher rank cases. This talk is based on the joint work with Professor Jurgen Berndt (University College Cork).

Keizo Yamaguchi (Hokkaido University)

"Parabolic Geometries associated with Differential Equations of Finite Type"

Abstract:

We present here classes of parabolic geometries arising naturally from Se-ashi's principle to form good classes of linear differential equations of finite type, which generalize the cases of second and third order ODE for scalar function. We will explicitly describe the symbols of these differential equations. The model equations of these classes admit nonlinear contact transformations and their symmetry algebras become finite dimensional and simple.

Title : 3-graded decompositions of exceptional Lie algebras $\mathfrak{g}$ of type E_7 and group realizations of $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed}$

Speaker : Toshikazu Miyashita

Abstract : The ν-graded decompositions of simple lie algebras $\mathfrak{g}$, $\mathfrak{g} = \sum_{k=-\nu}^{\nu} \mathfrak{g}_k$, $[\mathfrak{g}_i, \mathfrak{g}_j] \subset \mathfrak{g}_{i+j}$, has been studied by many mathematicians. Firstly, the case ν=1 was studied by S. Kobayashi- T. Nagano. The case of ν=2, S. Kaneyuki classified and determined the types of subalgebras $\mathfrak{g}_{ev}, \mathfrak{g}_0$ of $\mathfrak{g}$. I. Yokota and T. Miyashita gave group realizations $\mathfrak{g}_{ev}, \mathfrak{g}_0$ in the exceptional case. Reacently, for the exceptional Lie algebras $\mathfrak{g}$ of type E_7, M. Hara classified the 3-graded decompositions of simple Lie algebras $\mathfrak{g}$,

$$\mathfrak{g} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3$$

and determined the types subalgebras $\mathfrak{g}_{ev} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_2$, $\mathfrak{g}_0$ and $\mathfrak{g}_{ed} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_3$ of $\mathfrak{g}$. For the exceptional Lie group G of type E_7, we determine the strucure of groups for the subgroups G_{ev}, G_0 and G_{ed} of G corresponding to the subalgebras $\mathfrak{g}_{ev}$, $\mathfrak{g}_0$ and $\mathfrak{g}_{ed}$ of $\mathfrak{g} = \mathrm{Lie}\, G$.

Graded Lie Algebras, Geometric Structures and Differential Equations on Filtered Manifolds

Tohru Morimoto

A filtered manifold is a differential manifold M endowed with a filtration $\{\mathfrak{f}^p\}_{p\in\mathbb{Z}}$ consisting of subbundles $\mathfrak{f}^p$ of the tangent bundle TM such that

1. $\mathfrak{f}^p \supset \mathfrak{f}^{p+1}$,
2. $\mathfrak{f}^0 TM = 0, \quad \bigcup_{p\in\mathbb{Z}} \mathfrak{f}^p = TM$,
3. $[\underline{\mathfrak{f}}^p, \underline{\mathfrak{f}}^q] \subset \underline{\mathfrak{f}}^{p+q}$ for all $p, q \in \mathbb{Z}$,

where $\underline{\mathfrak{f}}^p$ denotes the sheaf of the germs of sections of $\mathfrak{f}^p$.

There is associated to each point x of a filtered manifold $(M, \mathfrak{f})$ a graded vector space $gr\mathfrak{f}_x = \bigoplus gr_p\mathfrak{f}_x$, where $gr_p\mathfrak{f}_x = \mathfrak{f}^p_x/\mathfrak{f}^{p+1}_x$, which then has a natural bracket operation induced from that of vector fields and becomes a nilpotent graded Lie algebra.

Under the slogan of nilpotent geometry and analysis we have been studying geometric structures and differential equations on filtered manifolds by letting the tangent nilpotent Lie algebras play the usual role of the tangent spaces.

In this talk I will discuss links between Lie groups (Lie algebras), geometric structures and differential equations from the viewpoint of nilpotent geometry and analysis. In particular, I will present a simple principle to associate to each representation of a graded Lie algebra differential equations on filtered manifolds endowed with geometric structures of which type is assigned by the graded Lie algebra.

Orbits of differential equations under actions of diffeomorphism groups

Hajime SATO (Nagoya University)

We can express the orbit of the system of PDE

$$\left\{ \frac{\partial^2 y_k}{\partial x_i \partial x_j} = 0 \quad (1 \leq i \leq j \leq m,\ 1 \leq k \leq n) \right\}$$

under the action of diffeomorphism groups of $\{x_1, \ldots, x_m, y_1, \ldots, y_n\}$, or

$$\left\{ \frac{\partial^3 y}{\partial x_i \partial x j \partial x_k} = 0 \quad (1 \leq i \leq j \leq k \leq m) \right\}$$

under the actions of contactomorphism groups of $(x_1,..,x_m, y, p_1,..,p_m)$, by using Schwarzian derivatives or contact Schwarzian derivatives.

In this talk, I will discuss the problem for the wave equation

$$\frac{\partial^2 z}{\partial x \partial y} = 0 \tag{1}$$

under the action of contactomorphisms of the space (x, y, z, p, q). This is an old problem of Lie. The orbit lies in the space of hyperbolic Monge-Ampère equations

$$\{Ar + 2Bs + Ct + D + E(rt - s^2) = 0 \mid \ B^2 - AC + DE > 0\}. \tag{2}$$

We aim to find a system of differential equations whose solutions give a contactomorphism mapping (1) to (2). This system of PDE turns out to be of the first order PDE with coefficients given by the coefficients A, B, C, D, E of (2). Thus it works in C^∞ category.

The integrability condition of the PDE is expressed by the coefficients A, B, C, D, E. If the condition is satisfied, we get the contactomorphism.

These are from joint works with Hiroshi Suzuki and Tetsuya Ozawa.