

Corrections to 'Spectral Theory and Differential Operators'

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correct9.pdf, September 2004

I am collecting errors and misprints for this web-page. Please send any you find to me at E.Brian.Davies@kcl.ac.uk

page 15 The end of the proof of Theorem 1.2.10 has a gap, which may be filled as follows.

For any $\phi, \psi \in \mathcal{H}$ the function $f : \mathbf{C} \rightarrow \mathbf{C}$ defined by

$$f(z) := \langle (z - H)^{-1} \phi, \psi \rangle$$

is entire with

$$|f(z)| \leq \frac{c}{|\operatorname{Im}(z)|} \quad (1)$$

for all $z \notin \mathbf{R}$. We now need to prove that this implies that f vanishes identically. This being so we derive the contradiction that $(z - H)^{-1} = 0$ for all $z \in \mathbf{C}$.

Theorem 1 *Let f be an entire function satisfying (1). Then f is identically zero.*

Proof Let $a \in \mathbf{R}$ and let n be a large positive integer. Define

$$g(z) = f(z) \frac{n^2 - (z - a)^2}{n^2}$$

and let γ be the circle with centre a and radius n . The second factor on the RHS of the above equation is bounded by 2 on and inside γ and vanishes at the points $z = a \pm n$, which lie on γ . An application of Cauchy's integral formula for derivatives yields

$$|f'(a)| = |g'(a)| \leq \frac{1}{n} \max\{|g(z)| : z \in \gamma\}.$$

Although the function g depends upon n and a , it is bounded on γ with a bound b that is independent of these parameters. From the bound

$$|f'(a)| \leq b/n$$

valid for all a and n , we deduce that $f'(a) = 0$ for all $a \in \mathbf{R}$. This implies that $f^{(m)}(a) = 0$ for all $a \in \mathbf{R}$ and $m \geq 1$, then that f is constant and finally that $f = 0$.

Another proof can be based upon the following facts. If H has empty spectrum then H^{-1} is a bounded self-adjoint operator with spectrum equal to $\{0\}$. By applying results about such operators we obtain the contradiction $H^{-1} = 0$.

p21 Exercise 1.13 should refer to the notation of Exercise 1.3, not 1.9.

p26,8 χ_m should be the characteristic function of $\{x : m < |x| < 2m\}$

p37,7 $L^2(S \times \mathbf{N}, d\mu)$

p37,14 $h \cdot U(\xi) \in L^2$

p38,-4 $= wr_w(H)U^{-1}(\xi) - U^{-1}(\xi)$

p42, (2.6.1) This is correct, but what is actually proved is the equally valid equality in which A and B are interchanged. Both yield the estimate of the lemma equally easily.

p61,1 Theorems 2.5.3 and 3.5.3

p65 Theorem 3.6.4 is often used in the form

$$\|f\|_p \leq c\|(H_0 + 1)^\alpha f\|_2$$

for all $f \in \text{Dom}((H_0 + 1)^\alpha)$, under the stated conditions on p , α , N .

p91,7 the norm of (4.4.2)

p101,13/14 Exercises 5.2 and 5.3

p104,12 an 14 $|x - y|^2$ not $(x - y)^2$

p104,-4 Exercise 5.6

p107,8 state explicitly that $s \in \mathbf{R}$

p112,10 constants missing on the RHS here and below

p113,-5 This should be

$$\begin{aligned} \int_{\Omega} |\Delta f|^2 d^N x &\geq \int_{\Omega} \{-\bar{f} \Delta f - f \Delta \bar{f} - |f|^2\} d^N x \\ &= \int_{\Omega} \{2|\nabla f|^2 - |f|^2\} d^N x \end{aligned}$$

p115, diagram $\phi(x) = \cos(x)$

p117 Problem 5.7 Hint: Use the geometry of the convex set to find a simple relationship between $d(x)$ and $d_\theta(x)$.

p119,-15 form of (6.1.1)

p125,3 The proof of this uses the bound

$$|p(x)| \leq c \operatorname{diam}(K_s)$$

for all $x \in \mathbf{R}^N$, which may be proved from page 124, line-2 by taking any y to be any point in the boundary of K_s .

p125,126 In five places here equations contain $N + \dots$ where they could contain $1 + \dots$. The equations are, therefore, correct as they stand, but there is no good reason for the change.

p127,-5 Corollary 4.2.3

p 128 Finish the proof of Corollary 6.2.2 with:
exists and is positive. The corollary follows by combining the inequality

$$M(\lambda_n - 1) < n \leq M(\lambda_n)$$

with the asymptotic expression for $M(\lambda)$.

p133,12 Script capital H not italic

p137,-7 is equal to

p138, Exercise 6.3 $\partial\phi/\partial x_r$

p143, proof of Theorem 7.1.4 It may be helpful to observe that differential operators of the form (7.1.2) can be characterized in a coordinate-free manner as those operators of the form $b_0 I + M$ where $b_0 \in C^\infty(\bar{\Omega})$ and $M : C^\infty(\bar{\Omega}) \rightarrow C^\infty(\bar{\Omega})$ satisfies the derivation property

$$M(fg) = M(f)g + fM(g).$$

p146, Theorem 3.6.4 with $\alpha = 1/2$ implies that if $N \geq 2$ and

$$f \in \operatorname{Dom} \left((H_0 + 1)^{1/2} \right) = W^{1,2}(\mathbf{R}^N)$$

then $f \in L^2(\mathbf{R}^N)$ whenever $2 \leq p < 2N/(N - 2)$.

p153, Exercise 7.1 This requires $N \geq 2$

p164 Theorem 8.2.8 Proof: One first chooses r, s and makes the orthogonal change of variables

$$y_r = (x_r - x_s)/\sqrt{2}, \quad y_s = (x_r + x_s)/\sqrt{2},$$

with $y_i = x_i$ for $i \neq r, s$. The relevant integrations are carried out for y_r before integrating the result with respect to all of the other components y_i of $y \in \mathbf{R}^{3n}$.

p176,8 the two eigenvalues

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