

Comments on and Corrections to “Science in the Looking Glass” commentary66.tex

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p2,-7 The fluid model of electricity is not simply folk-lore. It was proposed independently in 1747 by William Watson in London and Benjamin Franklin in Philadelphia. It had the important consequence of implying the conservation of the total quantity of electricity. Both took advantage of the recent invention of the ‘Leyden jar’ for storing electricity when conducting their experiments. Benjamin Franklin’s research culminated in his publication of ‘Experiments and Observations on Electricity’ in 1751. He was also responsible for proposing the use of lightning conductors to protect buildings.

p5,para2 Newton’s insistence on his corpuscular theory of light retarded the understanding of various diffraction phenomena for over a century, mainly because of his extra-ordinary reputation – Huygens’ wave theory of light, published in ‘*Traité de la Lumière*’ in 1678 faded from view. Even when Young provided more substantial evidence in favour of the wave theory of light in 1800, the response to his criticism of Newton’s work was largely hostile.

The situation only changed when Augustin Fresnel submitted a theory of diffraction to the French Academy in 1817. One of the judges of his submission, Siméon-Denis Poisson, sought to disprove it by using it to derive an apparently absurd experimental prediction about the transmission of light past and behind a small circular obstacle. Unfortunately for him, when the experiment was performed in 1819, the predicted effect was exactly as Fresnel’s theory predicted. At last the wave theory of light started to take over.

p5,5 This seems to suggest (wrongly) that a test cannot be taken seriously unless it is quantitative. I should not have conflated two facts: Lucretius had no serious tests of his theory at all, but the eventual proof of the atomic theory was in fact quantitative.

p5,para3 The three colour theory of vision was proposed by Thomas Young in 1802 and supported by further studies of Maxwell and Helmholtz later in the nineteenth century, but the three types of colour receptor in the retina were only identified in the 1960s.

p5,-15 ... amino acid change in the relevant protein. The procedure by which the genes (certain base pair sequences on DNA) produce proteins is described somewhat more fully on page 214.

p6,23 The list is below, not above!

p6,bottom There are many languages which do not distinguish between green and blue, Welsh sometimes being quoted as an example of this. While the Welsh word 'glas' used to carry this joint meaning, in present-day Welsh it is used for blue alone, and 'gwyrdd' is close in meaning to the English green. Whether this change was a result of the influence of English or of industrialization is hard to say. The assertions of Davidoff et al. about the Burinmo language are disputed by Paul Kay and others. See <http://www.icsi.berkeley.edu/~kay/> for further references to this controversial issue.

p7,figure 1.1 There should have been a spot marking the centre of the circle.

p11,figure 1.5 It is not clear whether there is a square or three rectangles in the centre of the figure. In fact it depends upon whether one focuses on the foreground or background.

p14,22 I have been criticized for using the pronoun 'they' as if it were singular – a recent way for avoiding the politically incorrect use of 'he'. I plead guilty. Language does change over time, but one need not hurry it along for reasons which may appear frivolous within a few decades.

p18,7 This is simplistic. Galileo pressed the idea in the quotation well before Hooke, and was as prolific an inventor of scientific instruments as Hooke. There were, in fact, instruments in existence at this time which measured quantities such as humidity and air pressure of which we have little or no direct knowledge. The mercury barometer was invented by Torricelli in 1643. In 1600 Queen Elizabeth's physician, William Gilbert, published 'De Magneta', devoted to an experimental investigation of magnetism, and in particular of the Earth's magnetic field. This was entirely dependent upon the construction of instruments to detect the direction of the fields.

p18,-12 Magee's comment that the process of repeated revision shows that thinking cannot be verbal in nature is not decisive. After observation of my own revision process, it seems to me that I have two distinct parts of my brain, one of which produces sentences, while the other assesses them for adequacy. The repeated

revisions might be the result of these two parts struggling to come to agreement with each other. This idea might even explain Specific Language Impairment and Williams' syndrome, mentioned on page 19. Introspection is not a way of settling how the brain assesses a sentence for adequacy, but Magee's argument about repeated revisions does not settle anything.

p19,8 Delete final comma.

p20,4 Cecilia Lai, Wolfgang Enard and others have evidence that one of the important genes in this context is one called FOXP2, which has a slight but crucial difference in humans and chimpanzees.

p20,6 I should have written 'as Chomsky used to argue'. His views have changed a lot since he wrote in 'Language and Mind' in 1968, about our being born with a system of several hundred rules for generating language, which underlie the actual languages which we speak. This notion has obvious resonances with AI, which was condemned as irrelevant to human understanding by John Searle. See p247.

p21,7 as well as the retention of others which would otherwise degenerate.

p24,11 Another example in which language does not fit reality is the classification of species, also discussed on page 218. Prior to Darwin the accepted (European) view was that species were rigidly distinct, each being created by God and given its own unique name by Adam (Genesis, ch. 2, v. 20). Darwin hoped that once it was fully appreciated that in many cases there were no clear distinctions between varieties and species, the urge to name and separate species might eventually recede into the background. Unfortunately the urge to classify in ever finer detail is not so easy to quench: there are today still arguments about the 'correct' definition of what constitutes a species. See: J LaPorte, Review, Brit. J. Phil. Sci. 54 (2003) 627-630.

p25,20 A supernova is not simply an explosion, but a complicated series of events starting with the gravitational collapse of the star's core at the end of its normal life cycle, followed by a huge explosion and finally a decay into quiescence as a neutron star or black hole, depending on its mass.

p25,25 Academics in this field do not claim to have *proof* of the existence of a black hole, only strong supporting evidence. See Science vol. 305, 27 August 2004, page 1238.

p27para-3 There are various other positions. One of the prominent ones is that the kind of mathematics which we have is largely determined by the nature of our brains, even though they evolved to the form they currently have for other reasons. In this view we could not help inventing numbers and then the other structures of arithmetic. While attractive, the long delay in the invention of negative num-

bers (and many other mathematical concepts now thought self-evident) does not support the idea.

p30,-15 Recent research has revealed the existence of an tiny Amazonian tribe called the Pirahã who are not able to distinguish between numbers bigger than about three. The adults are not able to learn how to count, but their children have no such difficulty. These facts provide strong support for the idea that counting is a cultural construct. See Science vol. 305, 20 August 2004, page 1093.

p30,l-9 On the positive side crows show real problem solving ability combined with the use and manufacture of tools. This is in spite of the fact that the most recent common ancestor of crows and humans was a small lizard-like animal living 300 million years ago. Thus behaviour that we recognize as intelligent has evolved independently at least twice.

p44,1 Descartes'

p44,-3 delete colon

p46,para2 Skinner was a highly influential American psychologist in the middle of the twentieth century, who did many experiments with rats and pigeons. He advocated the behaviourist view that animal and human nature was based upon responses to external stimuli, conditioning and reinforcement. Goals and free will were said to be irrelevant and unscientific concepts. In more recent times his reputation has plummeted. His views have been derided by some as extreme social Darwinism, of the type which Hitler espoused. All this indicates how immature psychology was as a science.

p46,13 some of our

p47,18 as suffering

p50,-9 I could have made the inadequacies of present day computers more obvious by asking for a verification that this inequality holds for $n = 10^{100}$, rather than a proof for all $n \geq 3$. Sometimes the general result is easier than the particular one! The inequality may be proved as follows. It is equivalent to

$$(n + 1)^{1/(n+1)} < n^{1/n}$$

Taking logs, this follows from the statement that $x \rightarrow \log(x)/x$ is a decreasing function for large enough x , which can be proved by using the calculus.

p51,15 Feng-Hsiung Hsu, who was deeply involved in the Deep Blue project from beginning to end, went even further when he wrote that the match was never really 'man versus machine' but rather 'man as a performer versus man as a toolmaker'. He, of all people, should know!

p52,para2 The self-esteem of some news commentators took a blow when it was revealed in 2001 that the human genome contains only twice as many genes (about 30,000) as those of the fruit fly and nematode worm. It appears that many of the extra genes are control genes, working in hierarchies. In computer terms the difference between a worm and a human depends much more on the sophistication of the program than on its size. See: J Sulston and G Ferry, *The Common Thread*, Corgi Books, 2003, p279.

p52,15 The figure of 98% is too low, but also obsolete. In September 2005 the journal *Nature* announced a draft genome sequence for the common chimpanzee. It is now possible to compare the human and chimpanzee genomes in such detail that summarizing the results in a single percentage has become a simplistic irrelevance. However, understanding how the differences between the two genomes leads to the different characteristics of the two species will need a huge further effort.

p52,18 Perhaps the word ‘radically’ should be deleted. Only time will tell.

p53 The literature on Penrose’s books and ideas is huge, and can be sampled by perusing Volume 2 of the journal *Psyche*. The article: T Maudlin, *Between the Motion and the Act*, *Psyche* 2(2) April 1995, draws an analogy between the functioning of the brain and the motion of the planets. The fact that planets do not have any computational powers does not prevent our predicting how they move by carrying out computations using Newton’s laws. Similarly our brains function the way they do not because they are following unconscious unknowable algorithms, but because they are physical objects and therefore subject to physical laws. We are capable of mathematical reasoning and communicate our results using language. That language is capable of logical analysis, but there need not be any *logical* connection between the way we think and the mechanisms we use to think.

p53,-8 The following quotation from Drew McDermott

<http://psyche.csse.monash.edu.au/v2/psyche-2-17-mcdermott.html>

emphasizes the stark difference between the logical consistency of the rules governing the operation of a computer and the behaviour of a program run on that computer.

‘I could take my self-contradictory program, and alter its behavior slightly, so that in response to “The earth is flat,” it would say “False,” and in response to “The earth is not flat,” it would say “True,” whereas in response to all other inputs P it continued to respond “P and not-P.” Computers are not committed to inconsistency on all issues after revealing an inconsistency on one issue, any more than people are.’

p54,18 repeated ‘the most’

p54,19 areas

p58,-17 Delete 'at'.

p62.10 Rewrite as '... if some mathematicians believe...'

p62:Whole Numbers See my comments here on pages 71-78 concerning the literature on the distinction between different sizes of number.

p62,-10 The trite answer is that numbers represent collections of idealized objects, so that collections of particular objects are instances of those numbers. But when one asks what the negative number -7 represents, one sees that there is no clear answer, just a variety of different possibilities, such as owing seven units of money or moving from the origin seven units to the left. The meaning of the rules for multiplying negative numbers is even less clear. Further investigations of these questions lead one straight into the history of the subject.

The eminent mathematician turned computer scientist Jacob Schwartz wrote the following in his article **Do the Integers Exist? The Unknowability of Arithmetic Consistency** "It is an article of faith for most mathematicians that Peano's axioms for arithmetic are consistent, perhaps because (in a kind of Platonist view) they simply state truths concerning a set of objects, the integers, that somehow exist, even though they constitute an infinite collection. A well-known (but extreme) statement of this point of view is Kurt Gödel's 1961 'The modern development of the foundations of mathematics in the light of philosophy'; see his *Collected Works*, volume III, Oxford University Press, 1981. Taken in an extreme form, the opposed nominalist position holds that, since we can have no direct physical experience with any such infinite collection, or even with its remoter members, the most that can be said is that extensive work with these axioms, including studies of their relationship to other axiom families considered likely to be consistent, demonstrates their consistency experimentally. If this latter position is accepted wholeheartedly, one ought to ask how thorough the search for an inconsistency implicit in this work really is, how it can be made more thorough, and how thorough a search might be possible in the best of cases."

p63,16 This takes the year 2000 to be the first year of the new millennium. Whether or not this is the case is a matter of opinion!

p64,para2 Al-Khwarizmi lived from 780-850AD approximately, not as stated. Babylonian mathematicians also developed a positional system for representing numbers, but with a space to represent zero.

p64,para2 The Treviso Arithmetic, published anonymously in 1478, provides an important insight into the widespread adoption of Hindu-Arabic numerals for commercial calculations in Italy. But Sacrobosco had already introduced them in the

thirteenth century in his ‘Algorismus’, which was widely used for several centuries.

p64,-2 An experimental test of this statement by the children of a colleague led to the number 1,340,000, but the type of sugar was not reported.

p66,para2,3 It could be argued that there are two issues here. One concerns vaguely defined concepts (what precisely distinguishes a tree from a sapling?) and is discussed further on pages 80-81. It can relate to quite small numbers. What, for example, is the number of world class universities in the UK? The point I was making is that for sufficiently large numbers there do not exist *any* physical instances of such numbers of objects.

p66,-3 The practical impossibility of producing a macroscopic object containing an exact number of atoms has real consequences. The kilogram is defined as the mass of a certain platinum-iridium cylinder kept in a vault near Paris. It measures about four centimetres in each direction and there are about eighty copies held in various places around the world. Unfortunately when these are compared against each other it is discovered that their relative masses are changing by 20 or more parts per billion per century, possibly because of the absorption of impurities from the atmosphere. Presumably this also applies to the standard kilogram in Paris, in spite of the fact that its mass is exactly equal to one kilogram by definition!

These facts indicate that the number of atoms in each copy changes enormously during the same period. Serious attempts are being made to define the unit of mass in a more stable manner, but it will take several more years before they come to fruition. One of the ideas is to define the kilogram as a certain precise number of atoms of silicon, which can be produced in an extremely pure form because of its importance in computer chips. The extreme difficulty of the task is indicated by the fact that if the standard kilogram were to acquire an extra surface layer just one atom thick its mass would increase by about one part in 10^8 and the number of atoms in it would increase by about 10^{17} .

If one tries to construct an exact cube composed of pure silicon, one runs up against the problem that silicon is very brittle, and it is not possible to prevent damage to the edges and corners or to take it into account accurately enough by scanning the surfaces with an electron microscope. For this reason current researchers use extremely accurately machined spheres of silicon, in spite of the fact that this shape bears no relationship with the crystal lattice structure of silicon.

p69,12 The following beautiful example of this is a special case of a result of de Moivre in ‘Miscellanea Analytica’, 1730. It may easily be extended to rewrite $x^{2n} + 1$ as the product of n real quadratic polynomials.

$$\begin{aligned}x^4 + 1 &= (x + e^{\pi i/4})(x + e^{-\pi i/4})(x + e^{3\pi i/4})(x + e^{-3\pi i/4}) \\ &= (x^2 + 2 \cos(\pi/4)x + 1)(x^2 + 2 \cos(3\pi/4)x + 1)\end{aligned}$$

$$= (x^2 + \sqrt{2}x + 1)(x^2 - \sqrt{2}x + 1).$$

The provides a purely algebraic solution to an algebraic problem by invoking both the complex fourth roots of unity and trigonometric functions!

commp71,-7 to p78 I did not invent the idea of distinguishing between accessible and inaccessible numbers. Under the name ‘feasible numbers’ the theory was invented by Rohit Parikh in 1971, and has been pursued by many others since that time. The theory may be developed formally in various ways, but the idea is that there are numbers which are so large that for they cannot be reached by counting within a feasible length of time. The function x^y , the result of multiplying x by itself y times, is highly suspect in this context, as anyone involved in numerical computations has known for a long time, and Parikh’s original paper focusses particularly on creating a ‘bounded arithmetic’ which avoids this problem.

If one accepts classical mathematics then one may prove (as a metatheorem) that some versions of Parikh’s theory are inconsistent, but the smallest chain of steps leading to a contradiction is so long that it is not feasible to write it down: it could not be done with all the resources in the universe. In other words the theory is consistent ‘for all practical purposes’. Actually one cannot prove that classical mathematics is consistent, so using it as the arbiter of whether feasible arithmetic is consistent is not obviously a good idea, unless one ‘knows’ that classical mathematics is consistent by some non-logical method.

p73,para1 More generally there is no practical algorithm for determining the first digit of an extremely large Fibonacci number; nor is it likely that such an algorithm will be found. This does not prevent the proof of some highly non-trivial statements about the set of all Fibonacci numbers. For example it has recently been proved that no Fibonacci numbers are powers (i.e. of the form a^b where $b > 1$) except for 1, 8 and 144. See Y Bugeaud, M Mignotte and S Siksek, preprint March 2004, which uses some of the deepest results in number theory to solve this famous problem.

p75, induction: Another example, which is strangely compelling because it is so simple, is as follows. Put n points anywhere on a circle and join them in all possible ways by straight lines. How many regions does the inside of the circle end up divided into? The first few cases seem to lead to a simple rule.

number of points	2	3	4	5
number of regions	2	4	8	16

The obvious inductive guess is that the number of regions doubles each time the number of points increases by 1. Unfortunately if one draws the simple diagrams involved one finds that for six points the number of regions is 31, not 32.

p82,6 Delete ‘brief’.

p83,[1]and[5] The reference became Davies 2003 after publication.

p85,-14 Most, not all. In 1955 Novikov and Boone proved that one of the central problems in group theory has no solution. The following very qualitative description is explained much more precisely in: Chandler and Magnus, The history of combinatorial group theory: a case study in the history of ideas, Springer-Verlag, 1982.

Groups are important mathematical objects, and come in many shapes and sizes. A particular group may look quite different from one viewpoint than from another, so it is no small problem to find out from given presentations of two groups whether the groups are the same or not. By using Turing's insights it has indeed been proved that there *does not exist any systematic method* of determining whether two groups have the same underlying structure (are isomorphic). In particular cases it may be obvious that they are and in others fairly easy to prove that they are not, but one cannot follow a prescribed procedure which is guaranteed to yield the answer. Because group theory is so central a subject in pure mathematics, the difficulty propagates to other subjects such as the impossibility of classifying manifolds in four or more dimensions. It is thought that three dimensional manifolds can be classified completely (up to homeomorphism) but this is not yet proved.

p87 In 2005 Georges Gonthier produced 'A computer-checked proof of the Four Colour Theorem', providing close to definitive evidence that the theorem is indeed correct. However, the proof is still not completely transparent from a human point of view. A S Calude has written a historical and philosophical survey of the development of the proof, published in the NZ Mathematics Magazine 38, 3 (2001), 27-35.

p91,para2 The fallibility of mathematics is well illustrated by the article of Michael Aschbacher in the Notices of the American Mathematical Society 51 (2004) 736-740. He admitted that a major hole in the proof, the classification of 'quasithin groups', had been discovered after 1980, and that it had taken seven years to close. He believes that no further important gap exists, but the complexity of the current proof of the Classification Theorem forces one to contemplate the possibility that there is still an error and that the accepted list of finite simple groups is not complete. More and more examples of theorems which take years to referee are appearing recently.

p92,-20 of a thousand

p97,[8] This story is repeated in many places, but the truth appears to be more complicated. It appears that de Moivre did not identify the constant in the equation as $(2\pi)^{1/2}$, and that this was Stirling's contribution. Whether this refinement is of major importance is a matter of judgement: I myself have seen mathematicians breaking off all contacts with each other over a dispute about which of them

deserves the main credit in such a situation.

p97,17and[9] It should be noted that Agrawal's algorithm is only fast in the technical sense of being deterministic polynomial. In practice it is much slower than the existing probabilistic algorithms. Nevertheless it is a major achievement.

p97 The most famous of all the hard problems is called the travelling salesman problem. It involves finding the *shortest route* starting and ending at one city, and visiting all of the others in a given region. No practical method of doing this is known – the idea of simply trying all possible routes and finding out which one is the shortest is wildly impractical except for small networks of cities. Many methods of finding a good (but not the best) solution within a reasonable length of time have been devised. One of the easiest to explain is the Ant Colony System, which models what real-life ants do when foraging for food. It was invented by Doriga and Gambardella in 1996. A large number of computerized ants try to find a route around the network, depositing a computer equivalent of pheromone at they go. As time passes the ants are attracted by the pheromone trails and tend to follow routes which have previously proved fairly successful. Eventually a local optimum is achieved. Clearly the program has to compromise between following the previous ant slavishly and exploring possible improvements to the route. Like most of the methods devised for such problems, this one has a random component, because the route followed by each ant is not completely determined.

p103 I should have mentioned that the symbol = was first used by the Robert Recorde in one of his many mathematics texts, published in 1557. Born in Tenby in West Wales, he went to Oxford at the age of fifteen and gained some prominence under the Tudor monarchs. He also made use of the newly introduced symbols + and –, which first appeared in print in a book on arithmetic by Johann Widmann, published in Leipzig in 1489.

p105,para1 How can one explain the difference between Euclidean and hyperbolic geometry on the one side and Riemannian geometry on the other? Describing the nature of their symmetry groups is not an option in a book of this type. It would need much more space than I allocated it. Also I should have deferred the discussion of Riemannian geometry to page 107.

p107,para2 The tools which Einstein used to construct general relativity were obviously taken from Riemann, but they needed adaptation: space-time does not have a positive metric unless one analytically continues to make time purely imaginary. Many researchers do precisely this!

p117,para3 Much simpler calculating machines had existed for many centuries. Pascal built several around 1650, while Leonardo (inevitably) designed one but did not build it. There is also a remarkable 'Antikythera mechanism' with 32

bronze gears, used for astronomical purposes and built in Rhodes around 80 BC.

p118,para-2 The key development which made them feasible was electronics, and in particular the triode vacuum tube, invented in 1906: even the earliest machines contained thousands of these. In the 1950s they were gradually replaced by transistors, whose operating principle was discovered in 1947, leading to much more compact and reliable machines.

p119,1 Turing was one of the intellectual leaders in the development of computers in the UK, but a very similar program was being followed in the USA, where John von Neumann was a central figure. Von Neumann was one of the most diverse intellectuals of the twentieth century, but he was first and foremost a mathematician.

page 120,para3 I completely missed a point of great interest here, that there is a time $T(n)$ such that every program of length n which does stop actually stops within the time $T(n)$. However, $T(n)$ is not computable, nor is any function bigger than $T(n)$ computable. See: Ruelle, Chance and Chaos p148 for the easy proof. One has to conclude that $T(n)$ increases incredibly rapidly.

p121,22 Replace by: The following are some of the considerations which show that this physical form of the thesis is *simply wrong*.

p127,-15 dais

p127,-2 If a scene

p129,-1 I should have added that there are equally strong arguments against this. Judges are drawn from a very small section of the population and may not always have the same values as those they are judging. It is tempting to claim that this cannot not matter when judging guilt or innocence, but I am less convinced about this than I used to be.

p131,para3 A short history of the slide rule may be found in Scientific American vol. 294, no. 5, p68-75, May 2006. It states that the first slide rules was made by William Oughtred in 1622 and the first pocket scientific calculator, HP-35, appeared in 1972. There is at least one book on the subject.

p134,7 number

p136,-11 This is not at all the way Bishop expressed it. He contrasted numbers which can be determined, or displayed, by a finite being, with those which only a being with non-finite powers, such as God, can discover. In my opinion bringing God into the picture only serves to confuse matters.

I do not agree with several other philosophical statements of Bishop, for example "We feel about number the way Kant felt about space. The positive integers and

their arithmetic are presupposed by the very nature of our intelligence and, we are tempted to believe, by the very nature of intelligence in general.” See Bishop, 1967, p. 2. This is extremely similar to a much earlier statement of Poincaré, whom I admire greatly. But Poincaré died before Gödel proved his theorems, which were to provide an enormous shock to the mathematical community.

I agree with Helen Billinge, who has analyzed Bishop’s arguments and finds them unpersuasive philosophically, although of real interest mathematically. See ‘Did Bishop have a philosophy of mathematics?’ *Phil. Math.* 11 (2003) 176-194. However, Bishop was trying to convince fellow mathematicians of the value of his ideas, not philosophers.

p137,para1 The following discussion of Fermat’s last theorem illustrates a key difference between Bishop’s constructive analysis and the traditional approach to the subject. One can replace FLT by a wide variety of other statements in number theory.

For every positive integer n define $F(n)$ by

$$F(n) = \min\{|a_1^m - a_2^m - a_3^m| : 3 \leq m \leq n, 2 \leq a_r \leq n\}$$

where all the variables are positive integers. $F(n)$ is a non-increasing sequence of non-negative integers, each of which can be evaluated in a finite length of time. FLT (proved by Wiles) is the statement that $F(n) \geq 1$ for all $n \geq 3$. The falsity of FLT would be the statement that $F(n) = 0$ for all large enough n . Traditionally it is accepted that every bounded monotonic sequence has a limit, but this amounts to the statement that FLT is either true or false. Traditionally trained mathematicians would also agree that $F(n)$ is eventually constant – there exists a smallest integer N such that $F(n)$ is constant for all $n \geq N$ – without this implying any method of determining or estimating the size of N (assuming that the truth of FLT is not known). Bishop took the position that such commitments did not carry the subject forward, and he rejected the use of the principle that every bounded set of real numbers has a least upper bound. Many mathematicians were surprised when he demonstrated how much analysis could be carried out by entirely conventional constructive methods without using it.

It is an interesting fact that by virtue of $9^3 = 6^3 + 8^3 + 1$ we see that $F(n) \leq 1$ for all $n \geq 9$. Since FLT is true we know that $N = 9$, but if FLT were false we would have $N \geq 10^6$, without knowing how large N was.

p137,5 According to a manuscript of Srinivas (The Methodology of Indian Mathematics), the Indian tradition of mathematics is much more in line with the constructivist philosophy. Much of Indian mathematics consisted of first describing a method of solving a problem, and then proving that the method led to the right answer in some generality. The proofs were often in detailed commentaries written by later authors. Srinivas (p18) wrote that the Indian philosophical tradition was unsympathetic to proofs by contradiction, particularly when used to show the

existence of a solution to a problem without finding it. In addition the tradition did not give primacy to set theory (p21) and did not make the close association between propositions and the set of all entities for which they are true, as Western mathematics has over the last century.

p138,6 ...looks at...

p140,19 This included Richard Feynman. See also page 169.

p140,para2 In a recent paper the author showed that standard quantum mechanics can be developed in a purely discrete space, and that the continuous world in which we appear to live might be the result of our only being aware of large scale phenomena. Even the familiar rotational symmetry of physical laws need not be present at the fundamental level. See E B Davies: Quantum mechanics does not require the continuity of space. *Stud. Hist. Phil. Mod. Phys.* 34 (2003) 319-328.

p140,28 Wolfram published his ideas in 'A New Kind of Science', 2002. This claims to herald a revolution in the way we look at the universe, which will supersede 'traditional science'. It should be said that most initial reactions to his book have been rather negative, although some of them have been written by people who admit that they are not a part of the generation of scientists weaned on computers. See for example: Steven Weinberg, 'Is the universe a computer?', *New York Review of Books*, 49, no. 16 (2002). It appears from the reviews that the book does not provide an explanation of any physical phenomenon which did not already have an explanation. The same applies to string theory, which has far more backers.

p141,12 Davies [2003]

p144,-20 I failed to understand an issue of major importance, emphasized in a recent article of G Goldoni: Copernicus Decoded, *Math. Intelligencer*, vol. 27, no. 3, 2005, pp. 12-30. The most important contribution of Copernicus, of which he was well aware, was to provide a method of evaluating the mean distances of each of the planets from the Sun (as multiples of the Earth-Sun distance). The Ptolomaic system was quite unable to do this, since it involved a method of description which would be unchanged if any of the heavenly spheres doubled in size, while the others stayed unchanged; in other words there was no way of incorporating scale information into it. Indeed Ptolemy admitted as much, and discussed disagreements between astronomers about the relative sizes of the spheres of the Sun, Venus and Mercury. When Kepler later derived his law relating the orbital diameters to the periods of the orbits, he *had to* base this on Copernicus's theory.

p144,-16 Replace 'provoke outrage' by 'not be well received'.

p144,-13 The story that Copernicus was given a copy of 'De Revolutionibus' on

his deathbed is apocryphal: he probably never saw it in print. Its publication was managed by a friend and Lutheran theologian...

p144,-1 In 'Astronomiae Instauratae Progymnasmata', published in 1587/1588.

p145,para2 The absence of parallax was only one of the problems with Copernicus' theory. If the Earth moved within a sphere containing the stars, then it would sometimes be closer to a particular constellation and sometimes further away. One would therefore expect the apparent size of a constellation to vary from season to season. The absence of both of these effects supported the idea that the Earth was at the centre of the celestial sphere.

Brahe made a major effort to detect the motion of a supernova which he discovered and then observed in detail during the last two months of 1572, but without success. This, combined with his later observations of a comet in 1577, provided evidence undermining the belief in the immutability of the heavens, but it was to take time for this to be fully appreciated.

p146,10 that our Sun

p146,para-2 This paragraph comes perilously close to endorsing the 'orthodox' myth of the Scientific Revolution of the seventeenth century: that Galileo initiated a completely new way of studying the world, which overthrew centuries of obfuscation by 'scholastics', 'peripatetics' and others, replacing irrationalism and the subservience to tradition with the hard light of reason and experimentation. The myth about his singular genius was promulgated by his biographer Viviani, and survived right into the twentieth century. Indeed it was so strong that the few who dared to challenge it were ignored or even ostracized.

One aspect of this myth was the attribution to Galileo of discoveries by earlier, medieval scholars. A particular example of this was his purported experiments dropping weights from the leaning tower of Pisa to demonstrate the falsity of Aristotle's claim that heavier objects fell faster. Not only is there no reason to believe that he did this, but the experiment had already actually been performed by the sixth century AD philosopher John Philopinus in Alexandria. Philopinus concluded from his results that Aristotle's physics of motion was seriously flawed, and his commentary had a substantial influence over the following centuries.

Several other achievements attributed to Galileo were actually the work of others. This is far from saying that he did nothing of importance, but his work has to be considered as a part of a developing body of knowledge, not as a complete break with earlier ideas. In Principia Newton attributed the formula $s=at^2/2$ for the distance travelled by a body in time t under constant acceleration to Galileo, who, inevitably for his time, had adopted a geometric formalism. However, closely related results been worked out by the Merton scholars and Oresme in the fourteenth century, and their work had been widely circulated and acknowledged.

Interestingly when Newton came to criticize the application of the constant acceleration formula to a body falling from the Moon (the Galileo-Plato problem) he did not attribute the mistaken calculation to Galileo, but to Blondel. Only in 1693 did he finally refer to Galileo in this context; see Cohen-Whitman pp 143-147.

There is another sense in which the notion of a Scientific Revolution can be understood: that the seventeenth century (and some decades around that) was a period of unprecedented growth which took the physical sciences to a new plane. It might be regarded as the time when science became an organized activity, supported by the foundation of scientific societies such as the Royal Society and the regular publication of scientific journals.

It is easy to persuade oneself that there was an explosion of scientific knowledge during the seventeenth century, but proving this assertion should depend upon providing a metric for the growth of knowledge. We discuss two such metrics, both derived from the resources at St Andrews. This is a list of biographies and related material for mathematicians throughout the ages. Each biography is written to high academic standards, with full references. It is, of course, possible that the material is subject to some systematic biases, but the onus is on those who doubt its objectivity to criticize the selection of material and provide what they consider to be a more objective list.

Of course mathematics cannot be identified with science, particularly the biological sciences. However, the St Andrews lists contains Leonardo, Brahe, Kepler, Galileo, Newton, Huygens and many others whom one might think of as physicists today. It is not unreasonable to expect that the growth of physics, at least, matches the growth of mathematics to a substantial degree.

We present our analysis in tabular form. We have divided the time between 1450 and 1750 into fifty year periods. Under the heading 'people' we list the number of those taken from

<http://www-groups.dcs.st-and.ac.uk/~history/Timelines/index.html>

who were between twenty and seventy at the start of the period. In most cases this means that they made most of their contributions during the stated period.

Under the heading 'output' we list the number taken from

http://www-groups.dcs.st-and.ac.uk/~history/Chronology/1500_1600.html for the same periods.

The first row gives results for every fifty year period before 1450. The results are quite clear. The production of new mathematics started increasing rapidly towards the end of the sixteenth century, and then continued into the seventeenth century. By 1700 the level of activity was at least three times higher than it was in 1500. Of course, one can ask about the quality of this increased amount of work, and a systematic analysis of this might be possible.

<i>Period</i>	<i>People</i>	<i>Output</i>
< 1450	< 15	< 10
1451 – 1500	15	8
1501 – 1550	18	14
1551 – 1600	36	14
1601 – 1650	60	44
1651 – 1700	53	59
1701 – 1750	57	44

Table 1. The Growth of Mathematics

p147,21 Statements like this are often simplifications of a very complicated story. In 1571 the Oxford astronomer Thomas Digges published a posthumous book by his father which showed that the latter had invented the telescope twenty years earlier; see Gribbin, p15. But the invention was stillborn, so Lippershey and Galileo should still be given due credit.

147,-16 Galileo had sufficient political nous to realize that it might be to his advantage to name the four satellites of Jupiter the Medicean stars. The ensuing blaze of publicity resulted in his appointment as Philosopher and Mathematician to Cosimo II de' Medici, the fourth Grand Duke of Tuscany, within a few months!

p148, para1 A lengthy discussion of the inadequacies of Galileo's instruments and arguments was given by Paul Feyerabend in 'Against Method'. He argues, surely correctly, that the sceptics had every justification for being doubtful, even about the existence of mountains on the Moon. However, most scientists accepted Galileo's conclusions by the second half of the seventeenth century: it is surely not *simply* a happy accident that they did so, nor was it *merely* that social pressures led people to interpret facts the same way.

p148,-2 ...should exhibit phases which varied in a manner that was inexplicable within the scholastic tradition. If Venus shone by its own light, as was commonly thought, then it should always be spherical, and a crescent would never be observed. If, on the other hand, it shone by reflected light, then according to the Ptolomaic system it should *always* appear as a crescent, because it was supposed to move around the Earth at a level below that of the Sun. When the disc of Venus was the smallest it would be furthest from the Earth and should appear as a very thin crescent. In fact Galileo observed the exact opposite: when smallest Venus was almost spherical in appearance, exactly as the Copernican system predicted. This failure of the Ptolomaic system only came to light with the invention of the telescope, because Venus was simply too small for the naked eye to be able to estimate its size or to see any crescent at all.

The difference between theory and observation is revealed quite clearly in this example. Copernicus' theory provided a simple explanation for three observed facts about Venus: it exhibited phases in certain positions in the sky, but not in others; The apparent diameter of Venus varied by a factor of a little over six as it moved in its orbit; the angle between the Sun and Venus never exceeded 46° . All of these were explained by the theory that the Earth and Venus rotated about the Sun, and that the distance from Venus to the Sun was about three-quarters (actually 0.72) of the distance from the Earth to the Sun. Kepler performed this calculation using the data that he inherited from Tycho Brahe. These were, of course, not the only successes of Copernicus' theory! Nevertheless the distinction between the observations and the Copernican explanation for them was undeniable,

and it remained plausible for some time that a quite different explanation would soon be forthcoming.

p150, para2 A proof of the rotation of the Earth was provided by Léon Foucault in 1851, when he demonstrated the rotation of the plane of oscillation of a 67 metre pendulum in the Panthéon in Paris. Foucault's pendulum attracted great attention at the Paris Exhibition of 1851, and was eventually to become the title of a famous book of Umberto Eco. While Galileo could indeed have performed this experiment, he did not understand dynamics nearly well enough to have interpreted the results.

p150,para3 This was not an isolated oversight. Galileo believed that the natural motion of bodies was in circles, and that the motion of the Moon could not involve any forces acting at a distance. It was left to Descartes to make the first attempt at a *dynamical* theory, in which bodies interacted with each other by means of forces, and in which their free motion was in straight lines. Descartes' theory was also physically flawed and was overshadowed by Newton's later achievements. See: L Kvasz, The mathematisation of nature and Cartesian physics. *Philosophia Naturalis*, 40 (2003) for more details of Descartes' contribution, which is largely ignored in my book.

p151,6 See my comments on page 261,para 3 below.

p151,paras2,3 The importance of the relativity of motion was emphasized by Paul Feyerabend in 'Against Method'. People in Galileo's era did not find this an easy notion, and he had to persuade them to alter the conceptual framework within which nature should be understood. We are very fortunate in living in a world with easy air travel. Sitting in an enclosed cabin travelling smoothly through the air at several hundred miles an hour, the correctness of this idea seems quite obvious to us. But this is a function of our circumstances, not of our greater intelligence.

p152,-1 Explanations always have to be in terms of some commonly accepted language, and at that time the language was Euclidean geometry, and in particular the use of lines and circles. Introducing a new type of curve for one purpose alone was regarded as unsatisfactory because it was ad hoc. Following Newton, differential equations slowly took over the position which Euclidean geometry once held.

p153,-14 Newton's first law of motion was taken directly from Descartes' 'Principles of Philosophy' published in Latin in 1644. However, Descartes' ideas about the laws governing the collisions of bodies were both elaborate and wrong.

p153 **1660-1680** It would have been useful to have interposed a discussion of social changes in Britain in this period in order to set Newton's achievements in context. We attempt this here.

During the seventeenth century Britain experienced rapid urbanisation. Commercial and surveying needs led to a great increase in numeracy among the population.

The earliest banks in London were founded by goldsmiths, providing safe-keeping for people's assets during the Reformation, and leading to the rise of accountancy. The invention of accurate clocks by Huygens in 1657 enabled the urban population to organize their lives ever more efficiently. Accurate maps and clocks became important for international trade and shipping; note that the Greenwich Observatory was founded in 1675. All of these developments increased people's willingness to believe arguments which relied on the use of mathematical calculations.

Evidence for the intellectual ferment of the period is abundant. The British Library contains George Thomason's collection of about 22000 pamphlets and early newspapers dating from between 1640 and 1661. The diaries of Hooke and Pepys show London's coffee houses becoming steadily more popular as centres of intellectual and political debate. Robert Hooke's *Micrographia*, published in 1665, revealed an entirely new world and became a best-seller. The Royal Society, awarded its royal charter by Charles II in 1662, held weekly meetings from this date onwards.

The increasing acceptance of mathematical arguments and the rapid improvement of telescopes led to major advances in astronomy during this period. The inverse square law of gravitation was suggested by Hooke during the 1670s. Several of Newton's ideas about dynamics and gravity (but not their detailed mathematical development) were already summarized on the last page of: Robert Hooke, *An attempt to prove the motion of the earth by observations*. 28pp, London, 1674. The majority of this work described a sustained attempt to observe the parallax of a star in the constellation Draco as measured from different points of the Earth's orbit. As discussed below (p 261) Cassini determined the distance from the Earth to the Sun in 1672, thus fixing the scale of the Solar System. Newton hammered the nails into the coffin of the heliocentric theory in 1687, but it was already well and truly dead in the minds of most scientifically educated people.

The widespread acceptance of the new model of the Solar System may be illustrated by a calculation of Olaus Roemer in 1675. He was observing the motion of one of the satellites of Jupiter, called Io, over several months, and noticed that the orbit was not completely regular. There was an anomaly of several minutes from one season to another. Roemer adopted the conservative approach of trying to find an adjustment to the model that would account for the discrepancy, and hit upon the idea that it might be due to the finite speed of light. This would imply that the orbital period of Io would appear to be slightly shorter when the Earth was moving towards Jupiter and longer when it was moving away from Jupiter. Roemer did not explicitly state what he considered the speed of light to be, but calculations based on his observations led to a moderately realistic value.

The interesting thing about this episode is that the Copernican model of the Solar System was so well established by 1675 that Roemer could use it to settle another controversial question. His work was published in the *Journal des Scavans* in 1676 and reported in *Phil. Trans. of the Royal Society* No. 136; June 25, 1677. Ten years later Newton mentioned it as well confirmed in *Principia* (page625). How-

ever, not everyone agreed: Cassini in particular did not feel that the accuracy of the observations justified drawing such conclusions from them. Logically speaking he may have been right, but eventually Roemer was vindicated. For further information see

<http://dbhs.wvusd.k12.ca.us/webdocs/Chem-History/Roemer-1677/Roemer-1677.html>
http://www.eaae-astro.org/eaae/news13/13.htm#L3_7

p153,-2 This is controversial, and I should have written ‘...may have been meant...’ The truth of the matter will probably never be certain.

p154,2 Newton’s account of his theory of fluxions, written in Latin in 1670/71, did not use the symbol \dot{x} to stand for the derivative of x , although its eighteenth century translations into English did so. His first use of this notation was in 1691, *after* the publication of *Principia Mathematica*. There is no evidence that he originally worked out the ideas of *Principia* algebraically. See: J Fauvel, *Newton’s mathematical language*, from ‘Foundations of Newtonian Scholarship’, eds. R H Dalitz and M Nauenberg, World Sci. Publ. Singapore, 2000. Since subsequent developments in calculus have depended heavily upon the use of algebraic notation, Newton’s avoidance of this is proof of the importance of the contributions of Leibniz.

Leibniz’s research in calculus started in the 1670s and led to publications using his algebraic notation including the differential dx and the integral \int in 1684 and 1686. His ideas were pursued vigorously by himself and others on the Continent, while the development of the calculus in England lagged well behind, because of people’s adherence to the less helpful notation of Newton there.

p155,-6 Newton’s failure in this respect was not a minor matter. He hoped to use the motion of the Moon as a way of measuring longitude at sea, and put a lot of effort into trying to predict its motion sufficiently accurately for that purpose. He did not succeed, and the longitude problem was finally solved when Harrison produced a series of increasingly accurate chronometers between 1736 and 1764.

p156,-7 I should have commented more fully on the importance of comets for seventeenth century astronomy. In 1664 and 1665 two comets attracted much attention in Britain, and the newly formed Royal Society asked Wren and Hooke to report on their movements. In May 1666 Hooke put forward the theory that the phenomena of the comets might be solved if they were attracted to the Sun by a force which made them move in a curve around it, but he kept open the possibility that they moved in straight lines.

There was a prior question about comets: were they near Earth objects or much more distant? In 1677 Hooke emphasized out that no measurements had been able to detect cometary parallax even for the well observed comet of 1664. He correctly deduced that comets were considerably more distant than the Moon. However, in

the same article he also stated that he considered that they shone by their own light, not by the reflected light of the Sun. It was left to Isaac Newton point out (in *Principia*, Book 3, Corollary 1, page 894) that the variations in the brightness of comets at different points of their orbits were not consistent with their shining by their own light. As usual he provided so much detail that the question was effectively closed.

Hooke was to goad Newton into investigation of a comet in 1680/1681. Cassini, Flamsteed and Hooke were all very interested in explaining the motion of this comet and a later one in 1682 now named after Halley. Newton devoted much of his energy to solving the mystery, in his usual isolated and secretive manner. His success was in considerable measure responsible for his abandoning Descartes' idea that planets were carried on vortices in the ether. It led to the publication of '*Principia Mathematica*' in 1687, but only because of repeated pressure put on him by Halley.

p157 One understands famous scientists best by examining their errors, not their successes: the latter have become ingrained so deeply in our way of thinking that they do not seem as surprising as they should, while their failures demonstrate the degree to which their arguments were robust. Not all of Newton's astronomical studies were as successful as *Principia*. His value for the mass of the Moon was quite wrong, and led him to conclude that the average density of the Moon was almost double that of the Earth (it is actually significantly less dense). Newton engaged in adjustments to his calculations which were not quite reputable, in order to try to get them to fit the observations.

One result of his error was Halley's theory that the Earth must be hollow, with another sphere (or spheres) inside it, rotating at a slightly different speed. This was also an early attempt to account for the slow movement of the Earth's magnetic field, discovered by Gellibrand in 1634; it might seem strange but our own explanation is hardly less so.

Halley then went on to make the bizarre suggestion that the interior spaces were probably inhabited. Supporters of his ideas have kept appearing right up to the present, but with steadily less scientific credibility as time has passed.

p158,3 According to the Epilogue on Determinism by G Nickel to Engel-Nagel's book on semigroups, Leibniz preceded Laplace with this idea. Both were opposed to Newton's view that God's intervention was necessary to keep the Solar System running over a long time scale.

p158 I should have mentioned the book: D Ruelle, *Chance and Chaos*, chapter 5, which treats the same issues. He referred to the history of metaphysical determinism (predestination), and there is obviously a lot more which could have been said about this. Note that this concept predated Newton by centuries (Aquinas, Calvin).

p160,para1 The new theory of chaos was formulated and accepted by physicists in the 1970s, and depended heavily on the possibility of detailed numerical simulations. The story is told in Chapter 11 of ‘Chance and Chaos’ by David Ruelle, one of the main participants in the events.

p160 after para3 The question of the stability of the orbits of the major planets is an extremely hard one, which both troubled and defeated Newton. Laplace thought that he had proved it, but he neglected terms that are critically important over times of 100 million years or so. In 1989 Jacques Laskar in Paris published the result of extremely detailed calculations showing that the orbit of the Earth is indeed chaotic over such a timescale. This has implications for the climate at such times, since the Earth’s orbit controls the amount of radiation received from the Sun, and how it is distributed in different latitudes.

p163,para3 One might also mention: J D Norton, Causation as folk science, preprint 2003, which discusses a single particle sliding smoothly on a spherical dome. If started appropriately, it can reach the unstable top point of the dome in a finite time, after which its motion is not determined. This shows that Newton’s equations do not always have unique solutions. The mathematics is well-known, but the example is easy to visualize.

p164,4 It would be better to write: Newton’s laws of motion must be replaced by relativity theory.

p164,Relativity The history of the discoveries is told in the recent book: P Galison, Einstein’s Clocks, Poincaré’s maps: Empires of Time, 2003. It appears that Einstein’s preoccupation with the coordination of time signals can be understood in the light of the development of the railway system in the nineteenth century and by his work as a patent clerk in Bern, Switzerland. This included assessing inventions involving electrical time signals and clocks.

p165 to p166,16. This is badly written. I should have made it clear that the term ‘Newtonian mechanics’ here refers to the part of Newton’s theory not involving gravity, and that relativity theory means special relativity throughout. In his famous 1905 paper, Einstein starts by discussing the application of Maxwell’s electrodynamics to moving bodies. Although not mentioning their names, he mentions the failure of the Michelson-Morley experiment to detect the Earth’s motion through the ether as it travels around the Sun. He knew of this experiment by 1899 according to John Stachel, Physics Today (ref.?). It was one of the paradoxes which his theory of special relativity resolved. My reference to Riemann should not be there.

p166,para2. This paragraph is too brief. I start discussing general relativity without explaining the difference between it and special relativity. Einstein did not get

as far as trying to unify relativity with quantum theory. He was attempting to unify general relativity (gravity) with electromagnetic theory, a much more plausible task, but one which was still beyond him. His later papers in this direction were not well received, and he became increasingly scientifically isolated as he grew older.

p169,10 The expedition was led by Arthur Eddington, who became famous for his subsequent contributions to relativity and astrophysics. There were, in fact two simultaneous expeditions; the one to Sobral in Brazil provided more data because of the weather conditions at the time. Neither of the series of observations provided definitive support for Einstein's theory, and its more or less instant acceptance had more to do with people's expectations and his reputation than with the weight of the scientific evidence.

p171,para3 This is the place to mention, if I am ever to do so, that the first manned flight in a hot air balloon was over Paris, and took place in 1783.

p172, Kolmogorov's axioms: The language of probability theory is often used even when there is no randomness involved. We already encountered this when discussing the digits of π on page 92.

When I was younger, I used to memorize numbers by factoring them into primes. Over a period of years I noticed that in many cases a number possessed a larger prime factor than I expected. Eventually I formulated the following conjecture: there is a high probability that a randomly chosen number n contains a prime factor larger than its square root. For example

$$12573 = 127 \times 11 \times 3 \times 3$$

and

$$127 > \sqrt{12573} \sim 112.13.$$

The mathematical theorem related to this conjecture makes no reference to Kolmogorov. It states that the fraction of all numbers less than n which have a prime factor bigger than their square roots converges to $\log(2)$ as $n \rightarrow \infty$; see D H Greene and D E Knuth: *Mathematics for the Analysis of Algorithms*, Birkhauser, Boston, 1990. In particular about 69% of all numbers less than a million have a prime factor bigger than their square root.

Elaborations on this result imply that implementing the fundamental theorem of arithmetic – that every number may be written (uniquely) as a product of prime numbers (Gauss 1801) – is likely to be computationally infeasible for large numbers. If one randomly selects a number with ten thousand digits that it probably has at least two prime factors with more than a thousand digits, and nobody has any idea about how one could factorize such a number. If this problem is ever solved, it will give various security agencies severe headaches, since many 'trapdoor ciphers' depend upon there being no effective algorithm for solving this problem.

p172,-17. Laplace was indeed responsible for formulating the general version of the central limit theorem in: *Théorie analytique des probabilités*, 1812. However, the particular case of the limiting form of the binomial distribution had been proved in 1730 by de Moivre, a French Protestant forced to flee to England in 1685 because of religious persecution. He became a close associate of Isaac Newton and a Fellow of the Royal Society. The binomial distribution is relevant to the outcome of a large number of tosses of a coin, and hence of interest in games of chance.

p174,-14 Chernobyl was by far the most serious of the three, having huge economic consequences not only in the immediate surroundings of the Ukraine, but as far away as the Welsh mountains, where the lamb industry was badly affected for several years after 1986. Restrictions were still in force in a few areas fifteen years later.

The least serious of the three accidents was the partial melt-down of a reactor at Three Mile Island in 1979, which unfortunately happened just after the release of an anti-nuclear film about just such an event called 'The China Syndrome', starring Jane Fonda. No nuclear power station has been built in the USA since that date! I should not have given the impression that any nuclear accidents have yet caused large numbers of deaths. They have the potential to do so, simply because the quantity of radioactive material in a nuclear power station greatly exceeds that in an atom bomb. The accidents show that this potential cannot be considered purely theoretical.

p175,para2 This was written two years before the Spring 2003 outbreak of SARS, which led to the virtual closure of air traffic from China to the outside world, and a level of quarantine of air travellers never before contemplated.

p175, Paradox of the Children It has been pointed out to me that this has flaws. By 'if you have a daughter' I intended 'if you already have a daughter', but it could also mean 'if you ever have a daughter', with no implications at all about the present children. However, the stranger's intended meaning has no importance in this context. What matters is the interpretation put on her statement by the mother. If she has decided not to have more children then her final statement can only be interpreted in terms of her present children. We do not know her intentions, but could make an informed guess if we knew her nationality, cultural background and religion.

There are other hidden problems. For example, two identical twins always have the same gender, and this implies that the genders of two children are more likely to be the same than different (other things being equal, which they are not). If the woman lives in a society which practises female infanticide, then the probability that at least one of her children is a boy would be very high. If, as in the UK, parents with two girls are more likely to have a third child than parents with one child of each sex, this would also affect the calculation.

There is yet another problem with the example. The final comment of the mother is rather unusual in its form: one would expect her to reveal more about her children. When people conceal information, they often do it to mislead the listener, and if one can guess the reason they are doing this, it could well affect one's calculation of the probabilities. This depends very much on understanding the social context of the conversation, and is why statistics is such a hard subject.

p176 The three door paradox. I should have stated this more precisely on line 2 of the section: 'According to the rules of the game, whatever the contestant chooses, the host, who...'

p184,-10 Although Lavoiser was the person who swept away the phlogiston theory, he did not make his discoveries out of the blue, any more than Newton had. Throughout the eighteenth century people such as Joseph Black, Joseph Priestley and Henry Cavendish had been turning chemistry into a precise quantitative science with the aid of the balance and Fahrenheit's thermometer. The particularly simple laws governing the chemical interactions of gases lay at the centre of Lavoisier's treatise.

p184,-17 Replace 'their existence' by 'the existence of atoms'.

p185,8 Avogadro's work languished in obscurity until it was developed and brought to international attention by Cannizzaro in a conference in Karlsruhe in 1860. See Gribbin, p375.

p185,10 This is an oversimplification. Several earlier people had proposed such tables, but their efforts had been ignored or ridiculed. Mendeleev predicted the existence of three new elements to fill gaps in his table, gallium, scandium and germanium; all three were discovered over the next twenty years, confirming the value of his theory. He was criticized at the time for 'arbitrarily' re-ordering the elements in a few places, but history has proved that he was right to do so. See: Gribbin, *Science, a History*, p379-

p185,para2 The use of ideas from structural chemistry became more widespread in the second half of the nineteenth century, and was given a great boost by van't Hoff's 'La chimie dans l'espace' published in 1875. This went far beyond Dalton's first attempts, and considered molecules as three-dimensional structures, rather than representing them by two-dimensional diagrams. However, many of those who used it still regarded it as a computational tool rather than as a true representation of reality. Hermann Kolbe, thirty years more senior than van't Hoff, described the latter's pamphlet as 'totally devoid of any factual reality and completely incomprehensible to any clear-minded researcher'.

p185,para3. The English astronomer Norman Lockyer was responsible for the discovery of an unidentified yellow spectral line in the Sun's spectrum in 1868. In

1870 he (correctly) suggested that it was due to a new element, which he named Helium.

p192,22 .. nothing to do with the effect of an observation on the subsequent motion of a particle.

p193,4 The recent study of Helium excitations in several laboratories around the world is just one example of the predictive success of quantum mechanics. It involves the quantum version of the three body problem, since a Helium atom consists of two electrons which repel each other while both are attracted to a heavy central nucleus. From the early days of quantum theory it was known that Helium has a stable rest state together with a large (theoretically infinite) number of excitations with precise energies. At higher energies there are many further excitations which have fairly sharp energies but finite lifetimes. Eventually each one of them disintegrates: one of the electrons emerges from the atom and is lost, leaving the nucleus and the remaining atom as a positively charged ion.

The theoretical investigation of the unstable excitations (called resonances) involves a highly non-trivial procedure called complex scaling, followed by huge computations. One has to determine the eigenvalues of non-self-adjoint matrices in spaces of dimension up to half a million. The experimental investigation is almost as demanding. In a conference held in Dresden in January 2004 it was reported that these two independent studies were in agreement to an extent which astonished both parties. This is truly a triumph for standard quantum theory.

The effects discovered are not relevant in 'normal' contexts, since they are usually obliterated by interatomic collisions. Their value is rather that they provide a sensitive test of the adequacy of standard quantum mechanics. If theory and experiment had diverged significantly this might have led to the discovery of new physics, and even to a Nobel prize. The techniques developed to attack this problem are also likely to be of value elsewhere.

p193,18 Quantum measurement theory is a topic which has no real analogue in other subjects. The formalism for extracting predictions about the outcomes of experiments from the fundamental equations is highly non-trivial, and is the focus of most of the controversy in the subject. The issue is whether the formalism reflects something which is actually happening during the measurement or whether it is simply our way of relating to an incomprehensible micro-world.

p193,-3 Perhaps I should have written that the electron has one internal degree of freedom, described by a vector in a two-dimensional space.

p196,21 Aspect was only one of the authors of the papers concerned.

p197,para4 Stephen Hawking, for example, stated quite clearly in: Hawking and Penrose, *The Nature of Space and Time*, Chapter 7, that the divide between them

was basically philosophical. Roger Penrose emphasized the need for modifications of quantum theory to resolve what he considered to be unacceptable paradoxes, but Hawking was not concerned about the phenomena in question. Such issues cannot be resolved by an appeal to the experts: there are too many world class scientists on both sides!

p198,Reflections The ‘problem of molecular structure’ also relates to the way in which molecules are actually discovered or created in practice. Chemists do not simply write down the full quantum Hamiltonian and find its eigenvalues and eigenfunctions from first principles. Even the fullerenes, which only involve one element, carbon, were not discovered this way, nor was there any realistic possibility that they could have been. They were first observed in the laboratory, then their structure was worked out by guesswork and intuition, and finally the results were confirmed computationally. There remains the possibility that carbon has further undiscovered molecular configurations, possibly smaller than that of C_{60} . If so it will not be discovered by ab initio computations directly from the quantum Hamiltonian. Chemical intuition is not an optional extra!

Computational chemistry depends heavily on the use of the Born-Oppenheimer approximation, which is usually regarded as no more than a technical tool. But it allows chemists to incorporate their previous intuitions about the shapes of the molecules which they are studying, so they are *not* working from first principles. See R G Woolley, B T Sutcliffe: P,-O. Löwdin and the quantum mechanics of molecules, *Fundamental World of Quantum Chemistry 1* (2003) 21-65; and B Simon: Schrödinger operators in the twenty-first century, *Mathematical Physics 2000* (eds. A. Fokas, A. Grigoryan, T. Kibble and B. Zegarlinski), Imperial College Press, London, 283-288. His Problem 12 asks ‘Is there a mathematical sense in which one can justify from first principles current techniques for determining molecular configurations?’

The reader should not be misled into thinking that there is an important disagreement between Roger Newton and myself concerning chirality. In his very positive review he criticized an oversimplified sentence (on page 198, line -13), which was made more precise (i.e. corrected) in the footnote 16 on page 202. Correspondence shows that he also considers that if an energy level splitting becomes small enough in a quantum mechanical model, then one should regard the energy level as physically degenerate. He agrees that neither of these resolves the problem of explaining why chiral states exist, and also agrees that one should not look for an explanation by invoking the asymmetry of the weak force.

The following is a fuller account of the double well quantum oscillator. At an abstract level if a Schrödinger operator H is invariant with respect to a parity operator P satisfying $P^2 = 1$ then every eigenspace \mathcal{L} corresponding to an eigenvalue λ is also invariant with respect to P . If the eigenvalue is non-degenerate then the eigenvector ϕ is either even or odd with respect to parity, that is $P\phi = \pm\phi$.

We next describe a simplified model which shows some of the essential features of chirality. Let H denote the Schrödinger operator

$$Hf = -f'' + Vf$$

acting in $L^2(\mathbf{R})$, where the double well potential V is given by

$$V(x) = (x - c)^2(x + c)^2/4c^2.$$

This operator is considered as a highly simplified model of the motion of the nuclei of a two atom molecule in an effective electronic potential, with the centre of mass coordinates removed. It is invariant with respect to the parity operator associated with the symmetry $x \leftrightarrow (-x)$. For large c its eigenvalues occur in pairs $\lambda_{2n}, \lambda_{2n+1}$ where $\varepsilon_n = \lambda_{2n+1} - \lambda_{2n}$ is very small and $\lambda_{2n} \sim 2n + 1$. The eigenfunction ϕ_+ associated with λ_{2n} is even under the space inversion symmetry but the eigenfunction ϕ_- associated with λ_{2n+1} is odd.

Chirality is the phenomenon that in many situations one finds that the substance corresponding to the Schrödinger operator does not relax to its eigenstates, but is found in one of the two states

$$\phi_{right} = (\phi_+ + \phi_-)/\sqrt{2}, \quad \phi_{left} = (\phi_+ - \phi_-)/\sqrt{2}$$

where ϕ_{right} is concentrated around $x = c$ and ϕ_{left} is concentrated around $x = -c$. The smallness of the eigenvalue gap ε_n explains why such states might be stable over very long time periods once they have formed, but there is no general principle to explain why these two particular superpositions are favoured.

Mathematically chiral states are selected by the use of the Born-Oppenheimer approximation, which fixes the positions of the nuclei. In the second stage of the method (often omitted), approximations to the nuclear eigenstates may be constructed by considering the quantum mechanical motion of the nuclei in the effective electronic potential.

One might argue, as does Roger Newton, that one should ignore the tiny gap between the eigenvalues and treat the Hamiltonian as if it had a doubly degenerate eigenvalue. The question remains: what physical rule implies that the molecule should take one of the two forms ϕ_{left} or ϕ_{right} as opposed to any other choice from the continuum of states in the two-dimensional approximate eigenspace? Also why is one form preferred by Nature to the other?

The best answers invoke interactions of the molecule with the external environment. A recent paper of M Klussmann et al (Thermodynamic control of asymmetric amplification in amino acid catalysis, Nature vol. 441, June 2006, pp. 621-623) provides experimental evidence for exactly such an effect. The phenomenon should more precisely be called ‘amplification of asymmetries’, since it depends on the asymmetry of the initial conditions, and not of the physical laws governing the development of the system.

p200,-2 It is often stated that von Neumann believed that there is an objective (or physical) quantum state and that measurement changes it discontinuously, but Lon Becker has recently argued that this is a misinterpretation of his position; see L Becker, ‘That von Neumann did not believe in a physical collapse’, Brit. J. Phil. Sci. 55 (2004) 121-135. The decisive evidence in support of this is to be found on page 440 of J von Neumann, ‘Mathematical foundation of quantum mechanics’, Princeton Univ. Press, 1955. Here he provides a detailed mathematical model showing how an ‘instantaneous collapse of the wave packet’ in a quantum system I can be modelled by a unitary evolution taking place over a period of time in an extended system $I + II$, where II is a model of the measuring apparatus. If one accepts Becker’s thesis then von Neumann’s position is close to that of my book. Of course von Neumann’s position is not decisive: we have to make up our own minds about the best interpretation of the theory. But it does shift the onus onto those who wish to support the ‘physical state’ theory.

p201, para2 Here I am adapting the Bayesian approach from statistics to quantum mechanics. Bayesian methods were widely regarded as scientifically unacceptable before about 1970, because they were considered to introduce a subjective element into science, which was supposed to be objective and absolute. Over the years attitudes have changed, but only slowly and painfully. In quantum theory many people still cling to the objective notion of a quantum state: somehow it is supposed to be objectively attached to the particle or group of particles to which it refers, while at the same time it is only a mathematical entity within a theory that has no rigorous relativistic formulation. No matter either that this leads to paradoxes associated with mysterious influences (which cannot be used to transmit information) travelling instantaneously from one place to another when measurements are made. I fully expect the adoption of Bayesian ideas in quantum theory to be as painful and slow as it was in classical probability.

p205,para3 I missed the opportunity to write that Mary Anning’s working class background and being a woman prevented her gaining formal recognition, even though she was recognized as the leading authority on fossils. She died fairly young, but made a reasonable income selling fossils.

p208,para2 Late in 2003 Tom Vail published a book arguing that the Grand Canyon was a result of Noah’s flood. It appears that this type of fundamentalist doctrine has the support of President George Bush. For more detailed arguments against this view than those put forward in my book see Wilfred A Elders, Bibliolatry Revisited: a review of “Grand Canyon: a Different View”, 2003. Elders points out (again) that the mainline Christian denominations do not support the view that the Earth began in 4004BC. The ‘arguments’ of Vail involve rejecting painfully acquired scientific knowledge (about fossils, geology and radioactive dating methods) on the sole basis that it contradicts what creationists think the Bible requires one to

believe.

p208,para3 Although this was not intended, the figure of 570 million years might give the impression of more accuracy than is justified. The Cambrian explosion is considered to have started about 542 million years ago, but recent discoveries have pushed the origins of complex multi-cellular organisms back to 580-600 million years ago. For example the minute organism *Vernanimalcula*, with a bilateral body, mouth and anus discovered in China may date back to that period.

p211,4 The scale of the effort put into dating techniques is indicated by the fact that there are dozens of academic centres world-wide devoted to tree-ring research. The following is an illustration of the value of this research. In the middle of the second millennium BC there was a world-wide climatic event, which was recorded in China as 'yellow fog, a dim sun, then three suns, frost in July, famine, and the withering of all five cereals.' There is unmistakable evidence of this event in tree rings from around the world, and it was almost certainly caused by the catastrophic explosion of a volcano on the island of Thera in the Eastern Mediterranean. Tree ring dating, involving hundred of trees of several different species, has finally pinned down this event to 1627/1628BC. This predates the fall of the Minoan city of Cnossos in Crete by over a century, demolishing the attractive theory that the one event was the immediate cause of the other.

p211,-1 This is incorrect. Carbon 13 comprises about 1% of naturally occurring carbon and is stable. Carbon 14 is indeed radioactive, with a half-life of about 5730 years, as a result of which it only occurs naturally in extremely small quantities. After 57 thousand years only one-thousandth of any quantity originally present is still undecayed.

p212,bullet1 There is absolutely no evidence in support of this, and a vast amount of experimental and observational evidence against it. Of course, one could contend that some malicious and powerful spirit might have changed the rate of decay some time in the past, and also systematically manufactured evidence which indicated that it had not changed. There in no way of countering such claims, which can be used in any context and have no serious content.

p212,bullet4 It is not only possible that the rate of radiocarbon production has changed from time to time: the variations in the rate have been investigated in great detail, and are routinely compensated for when using radiocarbon dating. Thus the absolute dates obtained from tree rings from different parts of the world have been compared with each other and used to calibrate radiocarbon dates over the last ten thousand years. It is known that the main causes of these variations are changes in Solar activity and in the Earth's magnetic field.

It is worth noting that the Vatican approved the use of radiocarbon dating on the famous Shroud of Turin in 1988. The unambiguous conclusion of three independent

laboratories that the samples taken dated from around 1300AD may or may not be decisive for the Shroud itself, but none of those involved in the continuing controversy deny the legitimacy of radiocarbon dating in itself. The argument has been about whether whether the samples taken from the Shroud were typical and uncontaminated. It is astonishing that ‘conservation’ work on the Shroud carried out in secret in 2002 has destroyed much of the scientific evidence about its origins.

p214 para3 I should have emphasized more strongly the difference between Darwin’s theory and the synthesis of Darwinism with genetics, often called neo-Darwinism. This originated with the work of Mendel, followed by others in the first half of the twentieth century, but it was immeasurably strengthened by Crick and Watson’s elucidation of the genetic code in 1953. It is becoming increasingly obvious that small genetic changes can sometimes have quite significant effects of the phenotype of the organism, so in that respect Darwin’s views about evolution have had to be modified.

A particular case is the loss of armoured scales in some freshwater species of stickleback, traced to a single gene called *Eda* by Kingsley, Schluter et al.; see the journal *Science*, 25 March, 2005 for details. This example may well be more complex than it seems on the surface. Since most fish do not have armoured scales, they probably developed in the oceanic stickleback as a protection against predators. This may have taken a very long time and have involved a large number of genetic changes. If so, and if the *Eda* gene plays a critical role in initiating the activation of the other genes, then a small modification in *Eda* may stop the whole process. It may be better to say not that it causes the evolution of a new phenotype, but that a modification in its structure may cause the reversion to an earlier phenotype.

p214,-9 A typical gene consists of thousands of such subunits, often, but not always, placed consecutively on the DNA molecule.

p214,-2 Put briefly, the information in the sequence of base pairs on the DNA is transferred to messenger RNA. This is transported to a ribosome where it is read three bases at a time and used to assemble a chain of amino acids – twenty of these are used. The chain then folds itself into a complicated shape to form a protein. The process is very complicated and the cell machinery plays an active role in controlling it at every stage.

p216,para3 In ‘The Extended Phenotype’ Ch2,p21 Richard Dawkins defends the use of the term ‘a gene for X’ even though he agrees with my comments. He argues that the word gene should not be interpreted as narrowly as I have done: it can simply mean an inheritable tendency to have certain features or behaviours, whether or not this depends upon a single gene in the sense of a functional group of base pairs on a DNA molecule. It may be the case, as he says, that geneticists understand what they mean by this phrase, but almost everyone else is bound to

be led into confusion.

The claim that some men possess a gene for infidelity, for example, carries with it the suggestion that they should not be blamed for extra-marital affairs. Or at least it suggests that infidelity has a simple physical cause, rather than being the outcome of a complex set of influences, including family background, social circumstances, the behaviour of the other party in the affair, and possibly even religious beliefs. Such 'genetic' reductions of complex behaviours run counter to our traditional views about moral responsibility.

In spite of this, it now appears possible that there is quite literally a gene which controls the vasopressin receptors in the brains of voles, and hence their fidelity. It is possible to enhance pair bond formation dramatically in adult meadow voles by using a virus to transfer a certain gene into them, and to block pair bond formation in prairie voles by inhibiting the expression of the same gene. See *Nature*, 429, 17 June 2004, 754-757.

The discovery of a profound effect of a single gene on the social behaviour of voles does not imply that the same holds for human beings, but neither can we rule out the possibility. We may one day have to decide whether infidelity is a deficiency which one should seek to cure, or whether it is morally neutral. This would be the same type of debate as is already taking place for homosexuality. In neither case does the answer to the scientific question resolve the ethical one.

p216,-18 In December 2003 Allen Roses of GlaxoSmithKline admitted that fewer than a half of all patients derive any benefit from most of the sophisticated drugs which have been developed by the drug industry. This fact was long concealed from the general public, but was well known to the experts. It is not scandalous: a one in three chance of getting substantial benefit from a drug is certainly worthwhile for someone with a serious illness. Genetic testing may well help to make prescribing a less chancy business, and also identify those likely to suffer side effects from a drug.

p216,-16 be a consequence

p218 Parts of Darwin's theory were anticipated by others, and he acknowledged many of the earlier contributions. In particular, James Hutton devoted a whole chapter of 'Principles of Knowledge', published in 1794, to the process of natural selection, but he did not accept that this could lead to the transformation of species, and was a convinced deist. Many of these people came from Edinburgh, where Darwin was a medical student, and he might well have been influenced by the intellectual atmosphere of the Scottish enlightenment.

p219,para2 It has recently been shown that the variation of beak sizes is caused by the levels of activity of a single protein. See *Science* magazine vol. 35, 3 Sept 2004, page 1383.

p219,-10 In his autobiography Darwin freely admitted his debt to Malthus, stating that he read 'On Population' in October 1838. The extent of that debt is discussed in detail in Chapter 3 of: Antony Flew, Darwinian Evolution.

p220,para4 This example, like that of Darwin's finches on page 219, suggests an approach to evolution in which changes in the climate plays the key role in evolution. In several cases it seems that climatic changes lead to rapid and rather predictable responses by the species affected. The changes might be based upon the Darwinian selection of random mutations, but the driving force is climatic change. A mechanical analogy is that of a ball sliding in a large, shallow dish which is tipped extremely slowly one way then another. While Newton's laws determine the motion of the ball, a person who looks at it only occasionally need not know this; he need only observe quite correctly that it is always at the lowest point of the dish. The law which causes this is not as important as knowing the shape of the dish. If it is not concave but has several peaks, troughs and saddle points, then the ball will always be close to one of the troughs, but not necessarily at the lowest one. In this case only the history of the dish's movement would enable one to explain why the ball is at one position rather than another. If one regards the ball as representing a species and the bowl as representing a multi-parameter space of possible variations of the form of that species, then one can easily imagine how a species could split into two if the surface has a complicated shape.

This metaphor was given the impressive name of the 'epigenetic landscape' by Conrad Waddington in 1957, and is frequently mentioned in scientific articles. But it is only a metaphor. It may provide some degree of insight, but it is simply too easy and general to have much content. There is no substitute for the work involved in finding out what is driving evolution in particular contexts, such as that of the cichlid fishes. This takes sustained efforts by teams of researchers.

p221,-8 returning to the original condition

p222 Somewhere on this page I should certainly have explained the use of the word Neo-Darwinism to describe the new synthesis.

p222, -7 The existence of metamorphosis in insects proves that one cannot infer the form of an organism from its genes alone. A butterfly has exactly the same genes as the caterpillar that gave rise to it: the difference lies in how the genes are expressed. The point at which the caterpillar pupates depends on the activity of certain hormones, which change the pattern of gene expression. So the production of these hormones leads to a complete change in the appearance of the insect. The same happens with humans at puberty, but to a much smaller extent!

p224,line3 The discovery of a missing link in the development from fish to land animals is new evidence of macro-evolution – the 'fishopod' properly named Tiktaalik

was first identified in Nunavut, northern Canada, and was described in 'Nature' in March/April 2006. It possesses rudimentary limbs but not the fingers and toes of later true tetrapods.

p225 I did not discuss Michael Behe's "irreducible complexity" argument for intelligent design. This is unacceptable as science since it does not recognize that an IC system may have got the way it is by losing earlier components, and the ones that remain might have quite different functions from what they had originally. For example the cow's tail, mainly used for swatting flies, evolved from something once used for swimming! Irreducible complexity is different from being the creation of a cosmic designer because evolution does not only proceed by building up organs one step at a time. Pete Dunkelberg has refuted ID by discussing numerous biological examples in <http://www.talkdesign.org/faqs/icdmyst/ICDmyst.html> A devastating critique of Behe's views about evolution is contained in the verdict of Judge Jones in the Dover, Pennsylvania case on 20 December 2005. He makes it clear that IC is simply a pseudo-scientific veneer applied to what is clearly recognizable as a religious position.

The human eye is not an example of irreducible complexity. One only has to look at a medical textbook to see a large range of different defects of the eye what impair its function but do not totally prevent it. These include the loss of elasticity of the lens with age, macular degeneration, colour blindness, loss of night vision, defects of the iris, loss of binocular vision, and many more. All of these are serious conditions, but a reduction in function is a long way from total blindness. Arguments about the evolution of eyes based upon their independent appearance in many different species do not demonstrate how mammalian eyes did evolve, but they do provide some ideas about how they might have evolved.

If the IC argument is shifted from the eye to the biochemical details of the functioning of the retina, as Michael Behe does in his article 'Molecular Machines' then it is little different from all other discussions about cell function: we know very little about the historical processes by which biochemical reactions within cells evolved, so statements that something could not possibly have happened millions of years ago have little force. As best they simply discourage people from trying to find explanations, and at worst they declare that thinking about such problems is close to rejecting one's religion. In this case the onus is on the IC advocates to prove their case. They are stating that something is definitely impossible, while the evolutionists are saying that they hope to produce mechanisms by which it could have happened. Although current theories about possible evolutionary pathways for bacterial flagella are pretty tentative, they at least have the merit that they might be proved wrong, which the IC arguments cannot ever be if one listens only to those advocating them.

p227,15 Delete 'being'.

p228,para2 This is not only a problem for those who believe in guided evolution. One might say that evolution was essentially over two hundred million years ago, and that since that time only relatively minor changes of proportion have occurred. If one examines the skeleton of *Diplodocus* (dating about 150 million years ago) one finds astonishing parallels with our own skeletons. Here is a short list of common features for the rear limbs and backbone alone: a single backbone at the rear of the body cavity, made up of vertebrae; a rib cage attached to the backbone and surrounding the body cavity; the vertebrae are rigidly attached to a pelvis; each rear limb has three parts – a single bone closest to the body, then a pair of parallel bones, then five digits, each composed of several small bones. No doubt an expert could go on for pages listing more detailed parallels. The standard explanation for them is that all land animals with backbones have a common ancestor. Possibly this skeletal design proved superior to all others, and so drove them to extinction; alternatively it arose by chance but was an evolutionary cul-de-sac in the sense that no further changes of a fundamental nature were possible once it had become dominant. But this is a ‘Just so’ story: it might be correct but finding evidence will be hard.

p228,para2 It is not relevant to human evolution, but worth recording, that the oldest known insect fossil, named *Rhyniognatha*, is almost exactly 400 million years old.

p228,23 I was not intending to imply that no conclusions about the cause of the explosion of forms of life at the end of the pre-Cambrian era would ever be forthcoming. There is a plausible current suggestion that it is linked to the relatively sudden development of properly functioning eyes by the predatory trilobites around 543 million years ago, and the consequent huge increase in the competition for survival. See Andrew Parker, *In the Blink of an Eye*, Oxford Univ. Press, 2003. However, this idea is not without problems and is far from being the new orthodoxy. Creationists or those who support guided evolution can argue that the eyes of the trilobites were too optically efficient for such a primitive animal and appeared too suddenly: they support the design argument more than the evolution argument. But even if one supports the basic thesis of Parker, he does not give a convincing reason why the evolution of vision was delayed so long if it could evolve so rapidly and give such a huge advantage to animals which possessed it.

p229,para1 One can go on asking questions about guided evolution, now more commonly called intelligent design. If one looks at a whale, one can see many anatomical similarities to other mammals, in spite of the huge differences. Darwin conjectured that whales had readopted a life in the sea after mammals had established themselves as land animals, and he even proposed that whales had evolved from a bear-like animal. A series of papers in the 1990s established that the DNA of a whale is extremely similar to that of a hippopotamus; from the genetic point

of view these two animals appear to be more closely related to each other than either is to another animal. Creationists have no explanation of these discoveries other than the suggestion that God chose to do this – which hardly ranks as an explanation since it can be repeated whatever the facts turn out to be. Evolutionists would say that it may finally have solved Darwin’s problem, not in the exact way that he envisaged, but within the framework of his overall theory. Whales did not evolve from hippopotami, but both evolved from a common ancestor, which presumably spent much of its time in the water; it is believed that this common ancestor existed about 55 million years ago. New discoveries are still being made in this field, and the summary above may need some modifications.

Those supporting guided evolution would say that God guided and directed the gradual transformation of mammals into the ancestors of the hippopotamus, and then caused some of these to move back into the sea, lose their four limbs and evolve into whales. This is not intelligent design, but a bizarre reversal of direction that no intelligent human designer would make. If you want a whale, then make a whale: don’t take a hippopotamus ancestor and then radically reconstruct it, wiping out more than a hundred million years of previous evolution. Evolutionary theory accommodated such changes readily, because it relies upon the opportunistic gabbing of current opportunities rather than movement towards some future goal.

p229,13 Replace ‘any’ by ‘and’.

p230,para1 A distinction between physics and other branches of science is that physics claims to be universal, while the biological sciences and geology are particular. Chemistry occupies an intermediate position in being universal although most of the subject is only applicable in environments which are not too extreme. Copernicus’ heliocentric theory was particular, and is still accepted. Newton’s explanation was universal, and has now been superseded by general relativity.

p231,last bullet point In the ‘Intelligent Design’ trial in Dover, October 2004, the philosopher Robert Pennock described methodological naturalism as reflecting the practice of science, and not as being imposed on science. His comments certainly influenced the ruling of Judge Jones against Intelligent Design.

p231,-3 Evolution *can* actually be tested in laboratory experiments provided one uses bacteria, which reproduce and can therefore evolve extremely rapidly. Such experiments are hardly likely to convince creationists (who cares what happens to bacteria?) but the results are in line with Darwin’s theory. The experiments are even repeatable, because bacteria reproduce by cloning and can be stored in suspended animation. See P B Rainey and M Travisano: Adaptive radiation in a heterogeneous environment, Nature 394 (1998) 68-72.

p236,para2 Stephen Hawking was one of the strongest advocates of the ‘Theory

of Everything’ – until recently. In January 2003 he announced in a public lecture entitled “Gödel and The End of Physics” that he had changed his mind and was ‘now glad that our search for understanding will never come to an end’. Although he is among the best known physicists as far as the general public are concerned, his recantation will probably not harm his reputation.

p236,8 Perhaps the biggest problem is pinning down what different people mean by reductionism. There are several quite different definitions ranging from the simple statement that it is a strategy for understanding physics and related subjects which has worked well in the past, and therefore ought to be pursued, to the declaration that the world really is governed in every respect by mathematical equations – the Theory of Everything.

p236,para3 I should have described the progress towards a full understanding of the laws of Nature over the last hundred years, before jumping into a discussion of the future.

The first example of the unification of two areas of physics was probably Maxwell’s transformation of electricity and magnetism into a single theory of electromagnetism. It is not surprising that this led to the design of electric motors and dynamos, and to the electrical industry dependent upon them. What is surprising is that when he calculated the speed of waves travelling in the electromagnetic field, he found it coincided with the velocity of light. This explained a phenomenon which had not been involved in the unification program, and led fairly quickly to the development of radio, and then television and radar.

Subsequently Maxwell’s theory and Newtonian mechanics were unified into special relativity and quantum theory, and then finally into quantum electrodynamics. Much later this led to the unification of electromagnetism and the weak and strong forces into the so-called ‘standard model’, which accounts for much of what is known about the world, but not for gravity. The standard model constitutes the current consensus, and is based on notions of group representations and gauge invariance that go back to Hermann Weyl in the 1920s.

Over the last thirty years there have been major attempts to unify the standard model with general relativity. Some development of (ten-dimensional supersymmetric) string theory – which originated in 1977 – is the current favourite candidate for the TofE. While it has some extremely eminent supporters, there are physicists, including Sheldon Glashow, one of the 1979 Nobel Prize winners in Physics, who wonder whether it is science or philosophy, and regret the lack of interest in experimental results by its devotees. Certainly direct experimental tests of string theory are far beyond the horizon, and its main merit (if it ever stabilizes) might be that it unified the other theories into a single set of equations. But in the absence of any direct predictions, its mathematical beauty would not prove that it was a true

picture of reality. See Glashow: interview July 2003.

The current goal is a Theory of Everything based upon string theory. Several eminent enthusiasts for a string-based ToFE hope that once properly understood it will be possible to use it to calculate the masses and properties of all of the fundamental particles from first principles; see Edward Witten, Comments on String Theory, preprint 2002. This is very optimistic, and there are no convincing grounds for believing it. However, even if the ToFE implied only a few relationships between the many currently arbitrary ‘fundamental constants’, it would have proved its value.

Most of those who follow this program probably describe themselves as reductionists, but one does not need to do so. Even if one believes that mathematical theories of the world are no more than our way of systematizing our knowledge, unification is a very worthwhile goal. Again, even if one believes that mathematics must ultimately fail to predict the behaviour of highly complex open systems, that does not mean one should not try to find out how much it can deliver.

p237 In physics a theory should have three components, not always neatly separable: laws (often mathematical), a domain of applicability and procedures for extracting predictions from the laws. Some people might say that the final component need not present in very complex situations if the theory provides ‘understanding’. In the ToFE the second is supposed to be unnecessary and the last is almost entirely missing. Nobody claims that any special feature of the Milky Way, let alone of the Earth, will ever be deducible from the ToFE, so its content is interesting only for those who regard such matters as of no import.

p237,-2 One of the simplest examples of emergent behaviour is the existence of an infinite cluster in percolation theory above the critical probability. Simple diagrams explain this well. From the point of view of the individual lattice sites nothing happens: one has to introduce a global concept before one can start to discuss the profound change which takes place. Percolation theory has many physical applications, such as helping to understand the properties of concrete. It has even been suggested that it may explain why we have not yet been visited by intelligent aliens!

p239,-20 such as the

p241,-5 than it appears

p243,-3 take into account

p244,3 which have such

p244,14 The Navier-Stokes equation may well be derivable from atomic dynamics, but the procedure for doing so is so complex that nobody can yet compute the

freezing point of pure water from first principles, let alone the effect on the freezing point of adding impurities.

p244,-3 Self-organized criticality is just one aspect of the study of complexity and emergent behaviour. The Santa Fe Institute was founded in 1984 to study all aspects of this, just when the growth of computer power made large scale simulations of the phenomena practically possible. Some examples are mentioned on pages 254-256. The behaviour of colonies of ants and bees (p 55) may also be described this way. The key idea is of complex, organized behaviour arising by the interaction of large numbers of individually very simple entities. If one adopts the standards of a classically trained physicist, one would have to say that the subject has little theory and much hype, but its whole point is that it provides a new paradigm of science, in the sense of Kuhn, (p 271). It should be judged by its own standards, in which computer modelling is much more central than finding a set of mathematical equations to describe the results.

p249,19 marry and have

p251,8 Delete full stop.

p253,para 2 The second law of thermodynamics is also relevant here. Not only is it impossible to create energy from nowhere, but it is also impossible to convert low grade energy (heat) to more useful forms without a substantial external input.

p254,18 a creator

p254,-8 In the weak form the anthropic principle says very little. For example, as human beings we necessarily evolved on a planet whose temperature lies within the range at which water can exist as a liquid. Since we see several other planets in our own Solar System, and recently still more elsewhere, it is simpler to accept the location of our planet as an historical accident, not as chosen to enable us to exist. It is less easy to take the same attitude towards the Universe, since we have no other universes to compare it with.

p256,-4 Jesus Mosterin has also recently written an article 'Anthropic Explanations in Cosmology' criticizing the anthropic principle in considerable detail. See <http://philsci-archive.pitt.edu/archive/00001609/>

p257,-1 This is an example of Eddingtonian numerology. Such ideas go back to Kepler and the Pythagoreans. See <http://www.brantacan.co.uk/index.htm> for more examples of apparent order arising out of what are in fact random data. I have not distinguished clearly enough between this and physical fine tuning. The difference is emphasized in: Anthropic reasoning, by M A Walker and M M Ćirković, Phil. Sci. Archive, Oct 2003. This is a critical response to the article of R Klee in my book.

p258,para2 One might have thought that the calculations of Livio et al. would have settled the issue, but that would be too optimistic. Even though everyone seems to accept their calculations, physicists can still argue about whether the substantial changes in stellar dynamics caused by a one percent change in a fundamental constant is remarkable or not. Steven Weinberg, for one, is not impressed, but Martin Rees is.

p259,para1 The latest attempt to by-pass the anthropic principle supposes that there is a vast variety of universes each with different values of the fundamental constants, most of them extremely unlikely to lead to the evolution of life as we know it, but some perhaps more propitious to the development of intelligence. If our universe is just one tiny part of what Martin Rees calls the multiverse, then the need to invoke a deity or designer to explain our existence becomes much less acute. This is a respectable idea, even if the current evidence for it is not at all conclusive.

Although it seems that we can have no knowledge about the detailed structure of these other universes, this has not prevented some physicists and philosophers from speculating that there may be super-intelligences living in some of them, creatures so far in advance of ourselves that they might even be able to run computer simulations of our entire universe. If that is the case then we ourselves may not be real: we might be a simulation on a computer of such a super-civilization.

The problem with these fantasies is that there is no way for us to test them scientifically, and we have simply to wait until one of the god-like people who is running the simulation decides to break into the program to inform us that everything we thought we knew, including our beliefs about the existence of the multiverse, is simply a joke at our expense. Once again the fact that some speculation is logically possible does not imply that it has any scientific merits. The last ditch defence of this idea – that someone needs to set the agenda with daring speculations for future generations to pursue – is too self-congratulatory. Terry Pratchett has done this particular job much more memorably than the current generation of physicists.

p259,-6 This of course refers to David Hume, the eighteenth century Scottish philosopher. John Hume, who won the Nobel Peace Prize in 1998, was Irish.

p260,1-15 I was surprised to find that this parable also appears on page 63 of Bertrand Russell's 'The problems of Philosophy', 1912, but for chickens rather than pigs!

p260,-3 I should have written more about the literature on this. Following Hume's analysis the status of causation has been examined by many people. Most of the ideas here are also to be found in Chapter 6 of Bertrand Russell's 'The problems of Philosophy', 1912. If he were to be to be criticised, it would be for hoping too

strongly that logical analyses of the notions of truth, falsehood, belief, etc. would contribute much to understanding the issues. In fact they do not get one much further than the obvious fact that almost anything we believe may be wrong, and we have to make decisions in this knowledge. A topical example was the conviction of both politicians and intelligence experts in the UK in September 2002 that Saddam Hussein possessed weapons of mass destruction, considered only a year later to have been largely incorrect. Some philosophers have argued that the concept of causation is incoherent, partly because it does not relate to the mathematical formulations of physical laws, almost all of which allow the flow of time to be reversed. This ignores the deep philosophical puzzles associated with the second law of thermodynamics, and also assumes that mathematical models have precedence over any other kind of explanation. Another line of argument is that we have incomplete knowledge of the world and finite abilities, so we are justified in using (indeed have no choice but to use) cause and effect arguments as incomplete models, even if they do not relate to the deepest nature (ontology) of the world. See for example: Antony Eagle, Pragmatic Causation, preprint 2003.

p261,para1 Here I am referring to what John Stuart Mill called the ‘axiom of the uniformity of the course of nature’ in: *A System of Logic*. See also J D Norton, p17 of : *A little survey of induction*, 2003.

p261,para3 There are two other defences of the use of induction, namely that one should choose the simplest, or the best, explanation of the facts known. In either case there seems to be no universal rule for determining what is simplest, or best, but it is often clear within a particular context. Norton, loc cit, argues that universal, context-free rules of induction do not exist, and that each case must be considered on its merits. The following is an example supporting this view.

Early in the seventeenth century there were deep disagreements about the best explanation of the astronomical facts. Four descriptions of the Solar system co-existed, due to Ptolemy, Copernicus, Brahe and Kepler, but none explained why the orbits should be as they were supposed to be. Kepler’s theory was the simplest, but it was not trusted because it involved ellipses, which did not arise elsewhere in science, and he provided no explanation for their relevance. Descriptions in terms of circles, on the other hand, fitted in with a long tradition, going back to classical Greek geometry.

Although Newton’s theory provided the first theory of *why* planets moved as they did, the choice between the four descriptions had already been settled in the minds of many scientists long before his *Principia* was published. For example Descartes wrote privately to Mersenne in 1633 supporting the heliocentric theory, but decided against publishing his views because of Galileo’s troubles with the Church. During the seventeenth century discoveries about the scale of the Solar System, the moons of Jupiter and the rings of Saturn all led people to re-appraise the previous view that the Earth was ‘obviously’ the primary object in the universe.

At the start of the century Tycho estimated that the distance from the Earth to the Sun was about 8 million kilometres. In *Micrographia*, p237, published in 1665, Robert Hooke proposed a method of finding the distance to the planets by measuring the parallax between sightings taken at two distant points on the same meridian. He envisaged the two observers drawing small diagrams, or iconismes, of the positions of the fixed stars around the planet as observed using a powerful telescope. This method would avoid problems associated with refraction of light by the atmosphere. In 1674 he wrote that estimates of the Earth-Sun distance by astronomers varied between 6 and 100 million kilometres, but he favoured the higher figure. Possibly he did not know that in 1672 Giovanni Cassini (who had just become the director of the new Paris observatory) had made a major advance by using a parallax measurement of Mars to determine the Earth-Sun distance as 140 million kilometres, tolerably close to the correct value of 149.6 million kilometres; this distance is now called the Astronomical Unit. He must have known that John Flamsteed in England had obtained a similar value in October 1672, but their relationship was always strained, and he must have chosen not to refer to this. By settling the absolute scale of the Solar System, the measurements of Cassini and Flamsteed also implied that Venus and Mars were similar in size to the Earth, that Jupiter and Saturn were much bigger, and that the Sun was enormous. Since Cassini and Hooke had also observed the rotation of Jupiter in 1664/1665, why should the Earth not also rotate? Its special status had vanished.

For the Sun to orbit around the Earth once every day at a distance of 140 million kilometres would imply that it was moving at a ridiculous speed. This argument applied with even greater force to the stars, whose distance from the Earth was clearly far greater than that of the Sun, if they were in fact suns themselves, as was coming to be accepted by 1670. The assumption that the Earth was rotating was much more plausible, and, once this step was taken, accepting that it was orbiting around the vastly larger Sun was made much easier. Galileo put forward this very argument in ‘Dialogue Concerning the Two Chief World Systems, 1632’ (see *The Cambridge Guide to Galileo*, p235) but without the numbers to back it up in detail. In England William Gilbert had written the same in ‘*De Magnete*’ published in 1600 (the very year in which Bruno was burnt at the stake) without, apparently, any fear of repercussions. The idea goes back to Aristarchus in the third century BC, but people were even less ready to accept such ‘mathematical’ arguments at that time.

We see that the question about which was the best description of the observations was resolved, not by general philosophical arguments, but by very particular considerations: much more accurate measurements which made the Ptolemaic theory wholly *implausible*, even though they did not *prove* its falsity in any logical sense. Robert Hooke considered that the *proof* of the motion of the Earth around the Sun depended upon the detection of the parallax of distant stars when measured at different seasons of the year. In 1669/1670 he carried out some carefully de-

vised measurements of the star Gamma Draconis, concluding (incorrectly) that its apparent position did indeed vary very slightly. He wrote

'Tis manifest then by the observations of July the Sixth and Ninth: and that of the One and twentieth of October, that there is a sensible parallax of the Earths Orb to the fixt Star in the head of Draco, and consequently a confirmation of the Copernican System against the Ptolomaick and Tichonick.

Unfortunately his instruments were not accurate enough for the purpose in mind, and the observation of stellar parallax had to wait for Wilhelm Bessel in 1838. The Royal Astronomical Society rewarded him with a gold medal, but the motion of the Earth was by that time long regarded as settled. From 1887-1889 the measurement of parallaxes was transformed by the use of photographs by Charles Pritchard and then others.

There have also been proofs of the motion of the Earth via Newton's laws. See the comments on Foucault's pendulum on p150 and on Halley's comet on p157.

p262,12 It is surprising that general relativity was accepted so readily in view of the very sparse evidence for it. It took many decades to amass a convincing range of predictions which confirmed the theory, but there is now no doubt that it passes these tests. Among the most compelling is an extraordinarily precise explanation of the slowly decaying orbit of the Hulse-Taylor pulsar, a pair of neutron stars in close and very non-circular orbits around each other. An account of the discovery of this pulsar at the Arecibo observatory in 1974 and of its significance for general relativity may be found on the 1993 Nobel Prize lectures of Hulse and Taylor on the web.

p262,end One should separate two different strands in Popper's account of scientific practice. One is the essential role of hypotheses in scientific practice, about which he is surely right. The other is his advocacy of tests which might refute a theory in preference to those which merely confirm it: a theory may be confirmed many times although it is wrong, but a single refutation stands for ever.

Unfortunately for Popper, scientists use more subtle methods to decide whether they should try to confirm or refute their favourite theories. At the present time hundreds of millions of dollars are being poured into the construction of a 'Large Hadron Collider' at CERN, which may reveal the presence of the Higgs boson. Most theoretical physicists pray that it will indeed be found, and thereby confirm the essential correctness of the Standard Model – the picture of elementary particle physics which they have been building up for several decades. Otherwise all of their efforts will have been in vain, and they will have to rethink the subject in a very fundamental way. Of course the experiments may go either way, but this is always

true; the issue is whether they were right to design this type of experiment in spite of Popper's advocacy. My vote is that they know what they are doing.

p265,25 The scanning-tunnelling electron microscope, STM, invented by Gerd Binnig and co-workers in IBM in 1981, not only enables one to see individual atoms but to move them to chosen positions. IBM have a gallery of atomic level images, including one of their own initials, produced using a STM. The ability to perform such manipulations (which is not confined to IBM!) could be regarded as the final proof of the reality of the entities considered. See <http://www.almaden.ibm.com/vis/stm/atomo.html>

p269,6 I am distinguishing here between standard quantum mechanics and relativistic quantum field theory, a subject full of technical difficulties which are still unresolved in four-dimensional space-time.

p269,22 phenomenon

p269,23 The actual evidence includes detailed measurements of the variations in the cosmic microwave radiation and observations of the red-shifts of the most distant supernovae and galaxies. The theories devised to explain these are at present in a state of flux.

p273 Paul Feyerabend has written an even stronger criticism of all those who try to argue that there is a 'correct method' of pursuing science. His book 'Against Method', first published in 1975, should be required reading for those who think that scientists go about their business in an orderly Baconian fashion. Nor does Popper's falsificationist theory escape his censure. He devotes a final 'Postscript on Relativism' to dissociate himself from the quite understandable charge that he is a relativist for whom all views about the world are equally valid.

p274,para3 The British leader of the Human Genome Project, John Sulston, wrote in 'The Common Thread' (p59) that he had no concerns about being involved in what was essentially a Baconian project. The goal of the HGP consisted almost entirely of transferring information from human DNA to a computer data-base. Knowledge about human development and disease were expected to follow *after* examination of the results. At an earlier stage in his career Sulston had been involved in tracing the complicated developmental history of each of the 57 ventral cord neurons of the nematode worm *C. elegans*, another purely Baconian enterprise.

p275,-1 It does not help, however, when scientists have to contend with absurd fears raised by characters such as Prince Charles. In April 2003 he raised the spectre of the world being destroyed by self-reproducing nano-robots, and called for an enquiry into the dangers involved in this technology. (In the Independent on Sunday on 11 July 2004, he denied ever having used the phrase 'engulfed in grey goo' popularly ascribed to him.) This is pure science-fiction, far beyond

anything which scientists could do even if they were so minded. Self-reproduction is an extremely sophisticated characteristic of living organisms, which viruses can only achieve by taking over the reproductive machinery of far larger cells. The effect of this media frenzy is to divert attention from a serious and present danger: that biological infections such as influenza, AIDS and SARS might devastate the world's population. Charles has far more opportunities than most people to find out which of his concerns are well-founded and which are not, and should do so before speaking out.

p275,-1 Recent publicity about the so-called sudden infant death syndrome SIDS will no doubt be quoted as another case in which scientists got it seriously wrong. If one examines the ruling of the Appeal Court in the case of Sally Clark in April 2003 a rather different picture emerges. In the original case nine expert medical witnesses had given evidence. Most had emphasized how little was known about SIDS. They had stated that the deaths of the two children involved had several unusual features and that the cause of death could not be ascertained. The appeal was allowed mainly on the grounds that the pathologist involved had not revealed evidence which might have been considered relevant to the defence. Professor Roy Meadow was criticized for presenting misleading statistics in the trial, but the original trial judge had warned the jury against relying upon statistical evidence. The President of the Royal Statistical Society disagreed with the trial judge and with the Appeal Court judges about the value of statistics in such cases. In January 2002 he wrote to the Lord Chancellor stating that experts in statistical procedures should be consulted more frequently, and that Roy Meadow had claimed to have statistical expertise when he was in fact ignorant of well-known fallacies in the subject: one cannot assume that two or even three cot deaths within the same family is clear evidence for murder, because a genetic predisposition would imply that deaths of successive children were not statistically independent. The moral in this tale is not easy to extract. Lawyers have to rely upon the adversarial system of prosecution and defence. One of its weaknesses is that confident expert witnesses who paint a simple picture are more likely to convince a jury, and are thus more likely to be selected as witnesses on future occasions – whether or not their confidence is well-founded. The RSS considers that statistical advice should be sought when necessary, and that experts in other fields should be challenged by statisticians as well as by lawyers if they make statistical claims. But statisticians are so aware of the difficulties of their field that they may confuse juries to the point of complete paralysis. Society is much less willing to defer to experts than it used to be. On balance, this is a good thing provided the courts present all of the relevant considerations in a balanced manner. The judge's role is now much more critical than it was when experts simply declared how things were, and everyone took their word on trust. The media have a major role to play in this new situation, and it is clear that they are woefully unprepared for the responsibility.

On 6 August 2004 the General Medical Council had to rule against another medical expert involved in this case, Professor David Southall. According to them he had made a categorical judgement as an outsider when the facts in front of him did not justify it – he had made an assertion about the guilt of the father, which he later refused to qualify in any way, on the basis of watching a television interview! The GMC decided his actions were irresponsible and an abuse of his professional position, and suspended him from such activities for three years. It is worth noting that for many years before that he had been a highly respected expert witness in child abuse cases.

p275,para3 The conflicts between commercial organizations and the publicly funded groups involved in the Human Genome Project were often acrimonious, and were described at length in Sulston, *The Common Thread*. It is easy to take the line that knowledge should be freely available, and in this case I consider that this absolutely right. However, it is all too easy to condemn pharmaceutical companies for trying to protect the results of their research, and to forget that, if they cannot make a profit, the research will simply not be done and nobody will benefit.

p272,para 3 One might divide scientific revolutions into two broad types, while allowing particular revolutions to lie anywhere on the spectrum between them. At one extreme are theory-led revolutions of the type which Kuhn discussed, such as relativity and quantum theory, involving a radically new way of looking at the world. At the other extreme are revolutions resulting from the invention of new instruments, which make possible new types of investigation. Kuhn completely ignored these, although they have been just as important in the development of science. One example was the invention of photography, which led to the discovery of extremely faint cosmological objects by using very long exposures. Another was the Human Genome Project, which could not have happened without extraordinary rapid sequencing machines. As a final example note that *most* of what we know about the planets has resulted from sending space probes to examine them close up.

p275,-4 This issue was discussed at length in: *Meeting the Challenges of the Future, A Discussion between the 'Two Cultures'*, Balzan Symposium 2002, ed. W Rüegg, Leo S. Olschki Editore, 2003. The contribution of Lord Jenkin is particularly clear. The problems about communication and trust between scientists and the public concerning GM foods were emphasized by the reaction to the publication of a report on this commissioned by the UK Government in October 2003. Almost everybody applauded the parts of the report which fitted their own previous beliefs, and criticized as inadequate those which did not.

Robert May made one of the most outspoken speeches in this Symposium, in which he stated that the two cultures were not those of science and the arts, which were broadly in agreement. They were the forces of rationality and those of irrationality,

the latter often being associated with fundamentalist religious groups who wished to impose their doctrinaire views on the rest of the world. Needless to say, such groups were not very visible in the meeting.

p277,para3 The scientific revolution of the seventeenth century has many causes, and it is even argued in A Macfarlane and G Martin: A world of glass, Science magazine 305 issue 5698, 1407-1408, that the development of glass technology was very important.

p278,26 misplaced commas.