Science in the Looking Glass

What Do Scientists Really Know?

E. Brian Davies

Department of Mathematics
King’s College London
Almost every month some book or television programme describes exciting developments in cosmology or fundamental physics. Many tell us that we are on the verge of finding the explanation for the Big Bang or the ultimate Theory of Everything. These will explain all physics in one fundamental set of mathematical equations. It is easy to be swept along by the obvious enthusiasm of the participants, particularly when they are making real progress in pushing back the boundaries of knowledge. Unfortunately, most of their brilliant new ideas are doomed to be forgotten, if only because they cannot all be right.

Consider the currently fashionable idea that our universe is just one of many unobservable, parallel universes, all equally real. How can one hope to describe the inner structures of such universes, each with its own values of the ‘fundamental’ constants? Many may be dull and featureless, but others are presumably as fascinating and complex as our own. However much some physicists declare the reality of these other universes, in practice their main function is to support the mathematical models of the day, or to ‘explain’ certain properties of our own universe.

My goal in this book is not to adjudicate on the correctness of such new and speculative theories. We will instead consider the development of science in a historical context, in order to find out how such questions have been resolved in the past, and to explain why many long established ‘facts’ have turned out not to be so certain. My conclusion is surprising, particularly coming from a mathematician. In spite of the fact that highly mathematical theories often provide very accurate predictions, we should not, on that account, think that such theories are true or that Nature is governed by mathematics. In fact the scientific theories most likely to be around in a thousand years’ time are those which are the least mathematical—for example evolution, plate tectonics, and the existence of atoms.

The entire book is effectively an extended defence of the above statements. In the course of the discussion I risk the displeasure of many of my colleagues by explaining the feebleness of mathematical Platonism as a philosophy. I also provide psychological and historical support for the claim that mathematics is a human creation. Its success in explaining nature is a result of the fact that we developed much of it for precisely that purpose. Even the numbers which we use in counting become no more than formal symbols, invented by us, as soon
as they are as big as $10^{1000}$ (1 followed by a thousand zeros). Pretending that we can count from 1 up to such a number ‘in principle’ is a fantasy, and will always remain so. Moreover, it is not necessary to believe this in order to be interested in pure mathematics.

Whatever some over-enthusiastic physicists might claim, there is much which is beyond our grasp, and which will probably remain so. Subjective (first person) consciousness is one such issue. Understanding the true nature of quantum particles is another, in spite of the proven success of the mathematical aspects of quantum theory. Contingency, or historical accident, has obviously had a major influence on geology and biology, but some physicists think that it is even involved in the form of the laws of physics. Whether or not this is true, scientists are right to believe that, with enough effort, they can push the boundaries of their subjects far beyond their present limits.

An unusual feature of the book is that I try to explain why philosophical issues are important in science by means of simple examples. This is not the style followed by academic philosophers, but it makes the issues easier to understand, particularly in a popular context. In addition, discussions about the status of money, zombies, or rainbows are more fun than dry logical arguments about ontology.

I am painfully aware that the scope of the book is far wider than anybody’s expertise could span in this age of specialists. The attempt is worth making, because arguments informed by only one branch of science are inevitably distorted by that fact. I do not claim to have found the final answer to all of the deep questions in the philosophy of science, but hope that readers who have not previously thought much about these will see why they are important.

People vary enormously in their liking of mathematics. Many switch off as soon as they see it, and editors of popular books advise their authors to reduce it to the absolute minimum. I have gone as far as I can in this direction, and reassure the allergic reader that any difficult passages can be skimmed over. They are present to ensure that interested readers do not feel cheated by being told conclusions without any evidence in their support.

I wish to acknowledge invaluable advice, or sometimes just stimulation, which I have received from many friends and colleagues, in particular Martin Berry, Alan Cook, Richard Davies, Donald Gillies, Nicholas Green, Andreas Hinz, Hubert Kalf, Mike Lambrou, Peter Palmer, Roger Penrose, David Robinson, Peter Saunders, Ray Streater, John Taylor and Phil Whitfield. I do not, however, burden them with the responsibility of agreeing with anything I say here. I also thank my family for providing an atmosphere in which a task such as this could be contemplated; I know that the time which I have devoted to it has put me in great debt to them.
Contents

1 Perception and Language 1
   1.1 Preamble 1
   1.2 Light and Vision 3
      Introduction 3
      The Perception of Colour 4
      Interpretation and Illusion 6
      Disorders of the Brain 13
      The World of a Bat 15
      What Do We See? 16
   1.3 Language 18
      Physiological Aspects of Language 18
      Social Aspects of Language 22
      Objects, Concepts, and Existence 24
      Numbers as Social Constructs 27
   Notes and References 31

2 Theories of the Mind 33
   2.1 Preamble 33
   2.2 Mind-Body Dualism 34
      Plato 34
      Mathematical Platonism 37
      The Rotation of Triangles 41
      Descartes and Dualism 43
      Dualism in Society 46
   2.3 Varieties of Consciousness 49
      Can Computers Be Conscious? 50
      Gödel and Penrose 52
      Discussion 54
   Notes and References 59

3 Arithmetic 61
   Introduction 61
   Whole Numbers 62
Contents

Small Numbers 62
Medium Numbers 64
Large Numbers 65
What Do Large Numbers Represent? 66
Addition 67
Multiplication 68
Inaccessible and Huge Numbers 71
Peano’s Postulates 75
Infinity 78
Discussion 80
Notes and References 83

4 How Hard can Problems Get? 85
   Introduction 85
   The Four Colour Problem 87
   Goldbach’s Conjecture 88
   Fermat’s Last Theorem 89
   Finite Simple Groups 90
   A Practically Insoluble Problem 91
   Algorithms 93
   How to Handle Hard Problems 96
Notes and References 97

5 Pure Mathematics 99
   5.1 Introduction 99
   5.2 Origins 100
      Greek Mathematics 100
      The Invention of Algebra 103
      The Axiomatic Revolution 103
      Projective Geometry 107
   5.3 The Search for Foundations 109
   5.4 Against Foundations 113
      Empiricism in Mathematics 116
      From Babbage to Turing 117
      Finite Computing Machines 123
      Passage to the Infinite 125
      Are Humans Logical? 127
   5.5 The Real Number System 130
      A Brief History 131
      What is Equality? 134
      Constructive Analysis 135
      Non-standard Analysis 137
5.6 The Computer Revolution 138
Discussion 139
Notes and References 140

6 Mechanics and Astronomy 143
6.1 Seventeenth Century Astronomy 143
Galileo 146
Kepler 151
Newton 153
The Law of Universal Gravitation 154
6.2 Laplace and Determinism 157
Chaos in the Solar System 158
Hyperion 160
Molecular Chaos 161
A Trip to Infinity 163
The Theory of Relativity 164
6.3 Discussion 166
Notes and References 170

7 Probability and Quantum Theory 171
7.1 The Theory of Probability 171
Kolmogorov’s Axioms 172
Disaster Planning 174
The Paradox of the Children 175
The Letter Paradox 175
The Three Door Paradox 176
The National Lottery 177
Probabilistic Proofs 178
What is a Random Number? 179
Bubbles and Foams 181
Kolmogorov Complexity 182
7.2 Quantum Theory 183
History of Atomic Theory 184
The Key Enigma 186
Quantum Probability 188
Quantum Particles 190
The Three Aspects of Quantum Theory 192
Quantum Modelling 193
Measuring Atomic Energy Levels 195
The EPR Paradox 196
Reflections 198
Schrödinger’s Cat 199
Notes and References 202
8 Is Evolution a Theory?
   Introduction 203
   The Public Perception 204
   The Geological Record 205
   Dating Techniques 209
   The Mechanisms of Inheritance 213
   Theories of Evolution 217
   Some Common Objections 225
   Discussion 230
   Notes and References 232

9 Against Reductionism 235
   Introduction 235
   Biochemistry and Cell Physiology 238
   Prediction or Explanation 240
   Money 242
   Information and Complexity 243
   Subjective Consciousness 245
   The Chinese Room 246
   Zombies and Related Issues 248
   A Physicalist View 250
   Notes and References 251

10 Some Final Thoughts 253
   Order and Chaos 253
   Anthropic Principles 256
   From Hume to Popper 259
   Empiricism versus Realism 266
   The Sociology of Science 270
   Science and Technology 274
   Conclusions 276
   Notes and References 279

Bibliography 281

Index 289