

Platonism in Science and Mathematics

DMV Gauss Lecture

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A Proposition to be Discussed

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Even God is powerless to stop $2 + 2$ from being 4.

A Diplodocus



A Sense of Humour

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Were there rainbows a billion years ago, before any creature with eyes to see them existed?

Do they exist now?

Platonism and The Status of Possibilities

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That something actual turns out to be the mind of God.

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John Polkinghorne describes this as a metaphysical guess, which he does not accept.

I will next consider mathematical Platonism as described below. It is more than the statement that the mathematical consensus about something may be unrevisable in the assumed context.

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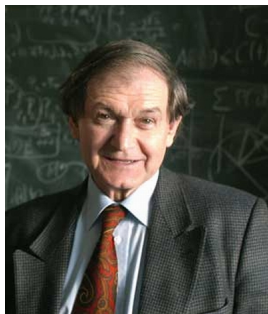
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In this view proofs are merely our way of ensuring that our dim perception of the truth is not misleading us.

The claim that one can make a logical distinction between the truth of a theorem and the existence of a proof of it is not self-evident, but it follows from Gödel's theorems *provided* one believes in the absolute Platonic existence of mathematical entities.



He has played a major role in the theory of black holes, quasiperiodic tilings, twistor theory.

When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of 'seeing' ... The mental images each one has, when making this Platonic contact, might be rather different in each case, but communication is possible because each is directly in contact with the *same* externally existing Platonic world!²

²Penrose, R. (1989). *The Emperor's New Mind*, p.428. Oxford Univ. Press.



In 1931 Kurt Gödel stopped the efforts to provide a firm foundation for mathematics by proving that in any sufficiently rich formal system there must exist a statement that cannot be proved or disproved within the system.

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It is often considered that Cantor and then Frege laid the foundations of a systematic theory of infinite objects, but the consistency of this theory is not known in spite of enormous efforts to resolve this problem between 1890 and 1930.

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Proposition 20, Book 9 of Euclid's Elements states that

Prime numbers are more than any assigned multitude of prime numbers

or, in contemporary language, 'given any finite list of prime numbers, there is another prime number not in that list'.

Euclid's algorithm

Euclid's proof was constructive: he described (rather sketchily) how to produce a new prime number from any given list. If one starts with the first few primes 2, 3, 5, 7, 11 then one may use them to produce the number

$$2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311.$$

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Mathematicians wave their hands when faced with such a problem and say that this does not matter because it could be done 'in principle'.

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- It fits in with the fact that mathematics seems to underlie all of our most successful physical theories.

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- There are innumerable cases in which mathematicians have agreed about some theorem only to have to admit that they should not have.
- No mechanism by which a Platonic world of mathematics could influence the physical world has ever been described.
The issue here is whether mathematics controls the motions of the planets or merely describes them.

Invention versus Discovery

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A drawing by Vincenzo Viviani in 1659

One could say that Galileo invented the pendulum clock or that he discovered the possibility of using pendulums to regulate clocks, and that this possibility had already existed before he discovered it.

Invention versus Discovery

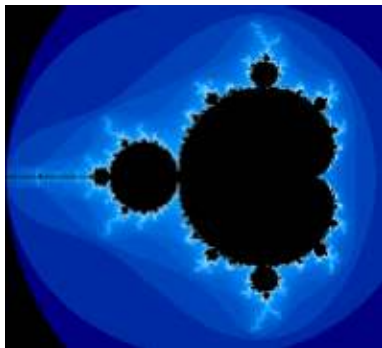


One might also say that stone-age axes were invented many millennia ago, or that our distant ancestors discovered that stones with sharp edges could be used for a variety of purposes, and then discovered that they could be made by hitting one stone with another.

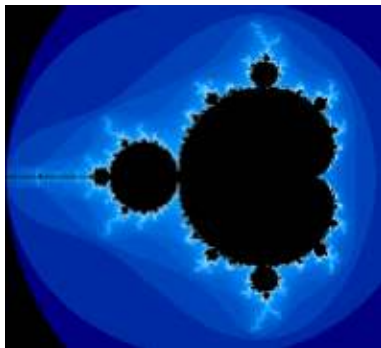


Mathematics is an evolution from the human brain, which is responding to outside influences, creating the machinery with which it then attacks the outside world. It is our way of trying to reduce complexity into simplicity, beauty and elegance. It is really very fundamental, simplicity is in the nature of scientific inquiry – we do not look for complicated things. I tend to think that science and mathematics are ways the human mind looks and experiences – you cannot divorce the human mind from it. Mathematics is part of the human mind.

The Mandelbrot set

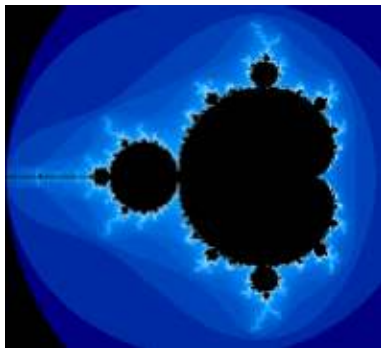


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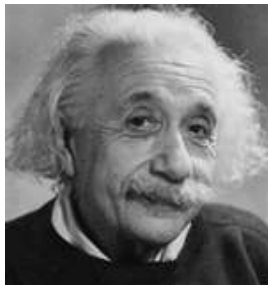


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An alternative view is that the possibility of obtaining very complex conclusions from simple assumptions has no particular philosophical significance.

Other complex structures

- The Japanese art of origami;
- The variety of different snowflakes;
- The number of games that can be played following the rules of Chess or of Go.
- The range of organic compounds based on carbon, hydrogen, oxygen and nitrogen;
- The huge variety of DNA molecules, all having the same fundamental structure;



The most incomprehensible thing about the universe is that it is comprehensible.

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However, mathematics is the most magnificent tool that our species has constructed so far and its power is not even close to being exhausted. We are right to celebrate that fact even if we are not Platonists.