Platonism in Science and Mathematics

DMV Gauss Lecture

E. Brian Davies

King’s College London

Jena, 7 May 2010
The problem with philosophical questions is that everyone knows the ‘right answer’ and almost nobody is willing to reconsider.
The problem with philosophical questions is that everyone knows the ‘right answer’ and almost nobody is willing to reconsider.

A mathematical truth (theorem) is discovered rather than invented. It is true before its proof has been found, before it is formulated, and would be true even if the human species had never existed.
The problem with philosophical questions is that everyone knows the ‘right answer’ and almost nobody is willing to reconsider.

A mathematical truth (theorem) is discovered rather than invented. It is true before its proof has been found, before it is formulated, and would be true even if the human species had never existed.

Even God is powerless to stop $2 + 2$ from being $4$. 
No diplodocus had a sense of humour.
No diplodocus had a sense of humour.

Some people might argue that the concept of a sense of humour existed as soon as the universe existed, even though it could not be applied to anything.
A Sense of Humour

No diplodocus had a sense of humour.

Some people might argue that the concept of a sense of humour existed as soon as the universe existed, even though it could not be applied to anything.

Were there rainbows a billion years ago, before any creature with eyes to see them existed?

Do they exist now?
(Neo-)Platonism and Christianity were linked by St Augustine.

1 God, Chance and Necessity, p. 36
(Neo-)Platonism and Christianity were linked by St Augustine.

According to Keith Ward, an eminent Oxford theologian

*Even if no actual universe existed, its possibility would exist, together with the possibilities of every other possible universe, all comprising an infinite set of possibilities. We are back to the Platonic world of pure forms, pure possibilities. But how can mere possibilities exist? One must be logically ruthless, and say that either there are really no possibilities or that they exist in something actual.*

---

1. God, Chance and Necessity, p. 36
(Neo-)Platonism and Christianity were linked by St Augustine.

According to Keith Ward, an eminent Oxford theologian

*Even if no actual universe existed, its possibility would exist, together with the possibilities of every other possible universe, all comprising an infinite set of possibilities. We are back to the Platonic world of pure forms, pure possibilities. But how can mere possibilities exist? One must be logically ruthless, and say that either there are really no possibilities or that they exist in something actual.*

That something actual turns out to be the mind of God.

---

1 God, Chance and Necessity, p. 36
One can imagine the possibility that green swans exist.
Green Swans

One can imagine the possibility that green swans exist.

One is not obliged to believe that there must exist a world in which the possibility of green swans is actualized, or that the possibility must already have existed in God’s mind in order for anyone to be able to entertain it.
Green Swans

One can imagine the possibility that green swans exist.

One is not obliged to believe that there must exist a world in which the possibility of green swans is actualized, or that the possibility must already have existed in God’s mind in order for anyone to be able to entertain it.
Platonism has a strong influence on theoretical cosmology.
The Multiverse

Platonism has a strong influence on theoretical cosmology.

According to many eminent cosmologists, including Martin Rees, there are indeed many different possible universes, each with its own values of the fundamental constants. Each of them really exists and they are all a part of the multiverse.

This is supposed to explain why carbon-based life exists in spite of its extreme improbability.
Platonism has a strong influence on theoretical cosmology.

According to many eminent cosmologists, including Martin Rees, there are indeed many different possible universes, each with its own values of the fundamental constants. Each of them really exists and they are all a part of the multiverse.

This is supposed to explain why carbon-based life exists in spite of its extreme improbability.

John Polkinghorne describes this as a metaphysical guess, which he does not accept.
I will next consider mathematical Platonism as described below. It is more than the statement that the mathematical consensus about something may be unrevisable in the assumed context. Theorems are supposed to be true statements about timeless entities, and to be true whether or not they have ever been or will ever be formulated by human beings. In this view proofs are merely our way of ensuring that our dim perception of the truth is not misleading us. The claim that one can make a logical distinction between the truth of a theorem and the existence of a proof of it is not self-evident, but it follows from Gödel’s theorems provided one believes in the absolute Platonic existence of mathematical entities.
I will next consider mathematical Platonism as described below. It is more than the statement that the mathematical consensus about something may be unrevisable in the assumed context.

Theorems are supposed to be true statements about timeless entities, and to be true whether or not they have ever been or will ever be formulated by human beings.
I will next consider mathematical Platonism as described below. It is more than the statement that the mathematical consensus about something may be unrevisable in the assumed context.

Theorems are supposed to be true statements about timeless entities, and to be true whether or not they have ever been or will ever be formulated by human beings.

In this view proofs are merely our way of ensuring that our dim perception of the truth is not misleading us.
I will next consider mathematical Platonism as described below. It is more than the statement that the mathematical consensus about something may be unrevisable in the assumed context.

Theorems are supposed to be true statements about timeless entities, and to be true whether or not they have ever been or will ever be formulated by human beings.

In this view proofs are merely our way of ensuring that our dim perception of the truth is not misleading us.

The claim that one can make a logical distinction between the truth of a theorem and the existence of a proof of it is not self-evident, but it follows from Gödel’s theorems provided one believes in the absolute Platonic existence of mathematical entities.
He has played a major role in the theory of black holes, quasiperiodic tilings, twistor theory.
When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of ‘seeing’ . . . The mental images each one has, when making this Platonic contact, might be rather different in each case, but communication is possible because each is directly in contact with the same externally existing Platonic world!

---

In 1931 Kurt Gödel stopped the efforts to provide a firm foundation for mathematics by proving that in any sufficiently rich formal system there must exist a statement that cannot be proved or disproved within the system.
Platonists believe that the infinite set of all natural numbers actually exists and has objective properties.

This is quite different from adding the existence of this set to one’s mathematical framework as a deliberate choice or convention.
Platonists believe that the infinite set of all natural numbers actually exists and has objective properties.

This is quite different from adding the existence of this set to one’s mathematical framework as a deliberate choice or convention.

It is often considered that Cantor and then Frege laid the foundations of a systematic theory of infinite objects, but the consistency of this theory is not known in spite of enormous efforts to resolve this problem between 1890 and 1930.
It is often said that Euclid proved that there is an infinite number of prime numbers, but this is a Platonic gloss on his actual result.
It is often said that Euclid proved that there is an infinite number of prime numbers, but this is a Platonic gloss on his actual result.

Proposition 20, Book 9 of Euclid’s Elements states that

*Prime numbers are more than any assigned multitude of prime numbers*

or, in contemporary language, ‘given any finite list of prime numbers, there is another prime number not in that list’.
Euclid’s algorithm

Euclid’s proof was constructive: he described (rather sketchily) how to produce a new prime number from any given list. If one starts with the first few primes 2, 3, 5, 7, 11 then one may use them to produce the number

\[2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311.\]

This is actually a new prime, but even if it were not it could be factorized to produce a new prime.
Euclid’s proof was constructive: he described (rather sketchily) how to produce a new prime number from any given list. If one starts with the first few primes 2, 3, 5, 7, 11 then one may use them to produce the number

$$2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311.$$  

This is actually a new prime, but even if it were not it could be factorized to produce a new prime.

Unfortunately from the point of view of computer science the algorithm is not feasible. It could not be used to produce the first prime larger than a billion.
Euclid’s algorithm

Euclid’s proof was constructive: he described (rather sketchily) how to produce a new prime number from any given list. If one starts with the first few primes 2, 3, 5, 7, 11 then one may use them to produce the number

\[2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311.\]

This is actually a new prime, but even if it were not it could be factorized to produce a new prime.

Unfortunately from the point of view of computer science the algorithm is not feasible. It could not be used to produce the first prime larger than a billion.

Mathematicians wave their hands when faced with such a problem and say that this does not matter because it could be done ‘in principle’. 
It corresponds to the way many mathematicians feel about their subject.
Merits of Platonism

- It corresponds to the way many mathematicians feel about their subject.
- It explains why everyone eventually agrees whether a theorem is true or false.
Merits of Platonism

- It corresponds to the way many mathematicians feel about their subject.
- It explains why everyone eventually agrees whether a theorem is true or false.
- It fits in with the fact that mathematics seems to underlie all of our most successful physical theories.
Research has shown that the way people feel their mental processes work bears no relationship with how they actually work.
Weaknesses of Platonism

- Research has shown that the way people feel their mental processes work bears no relationship with how they actually work.
- There are innumerable cases in which mathematicians have agreed about some theorem only to have to admit that they should not have.
Weaknesses of Platonism

- Research has shown that the way people feel their mental processes work bears no relationship with how they actually work.
- There are innumerable cases in which mathematicians have agreed about some theorem only to have to admit that they should not have.
- No mechanism by which a Platonic world of mathematics could influence the physical world has ever been described.
Weaknesses of Platonism

- Research has shown that the way people feel their mental processes work bears no relationship with how they actually work.
- There are innumerable cases in which mathematicians have agreed about some theorem only to have to admit that they should not have.
- No mechanism by which a Platonic world of mathematics could influence the physical world has ever been described. The issue here is whether mathematics controls the motions of the planets or merely describes them.
Invention versus Discovery

One could say that Galileo invented the pendulum clock or that he discovered the possibility of using pendulums to regulate clocks, and that this possibility had already existed before he discovered it.
One could say that Galileo invented the pendulum clock or that he discovered the possibility of using pendulums to regulate clocks, and that this possibility had already existed before he discovered it.
Invention versus Discovery

One might also say that stone-age axes were invented many millennia ago, or that our distant ancestors discovered that stones with sharp edges could be used for a variety of purposes, and then discovered that they could be made by hitting one stone with another.
Mathematics is an evolution from the human brain, which is responding to outside influences, creating the machinery with which it then attacks the outside world. It is our way of trying to reduce complexity into simplicity, beauty and elegance. It is really very fundamental, simplicity is in the nature of scientific inquiry – we do not look for complicated things. I tend to think that science and mathematics are ways the human mind looks and experiences – you cannot divorce the human mind from it. Mathematics is part of the human mind.
The Mandelbrot set

Penrose considers that the properties of the Mandelbrot set were discovered and existed independently of our becoming aware of them. An alternative view is that the possibility of obtaining very complex conclusions from simple assumptions has no particular philosophical significance.
Penrose considers that the properties of the Mandelbrot set were discovered and existed independently of our becoming aware of them.
Penrose considers that the properties of the Mandelbrot set were discovered and existed independently of our becoming aware of them.

An alternative view is that the possibility of obtaining very complex conclusions from simple assumptions has no particular philosophical significance.
Other complex structures

- The Japanese art of origami;
- The variety of different snowflakes;
- The number of games that can be played following the rules of Chess or of Go.
- The range of organic compounds based on carbon, hydrogen, oxygen and nitrogen;
- The huge variety of DNA molecules, all having the same fundamental structure;
The most incomprehensible thing about the universe is that it is comprehensible.
The real issue is not mathematics but the nature of the world. Mathematics is merely the name of our best current way of understanding certain simple aspects of the world around us.
The real issue is not mathematics but the nature of the world. Mathematics is merely the name of our best current way of understanding certain simple aspects of the world around us.

It does not explain ethics, our subjective consciousness and will not even enable us to predict the weather a month ahead.
The real issue is not mathematics but the nature of the world. Mathematics is merely the name of our best current way of understanding certain simple aspects of the world around us.

It does not explain ethics, our subjective consciousness and will not even enable us to predict the weather a month ahead.

What appears in journals is not mathematics. A research paper is text that stimulates us to think about some topic and tries to persuade us that a certain type of argument would lead to a certain result if it were written down in full. Understanding is possible because we have a certain type of brain and enjoy a common culture.
The real issue is not mathematics but the nature of the world. Mathematics is merely the name of our best current way of understanding certain simple aspects of the world around us.

It does not explain ethics, our subjective consciousness and will not even enable us to predict the weather a month ahead.

What appears in journals is not mathematics. A research paper is text that stimulates us to think about some topic and tries to persuade us that a certain type of argument would lead to a certain result if it were written down in full. Understanding is possible because we have a certain type of brain and enjoy a common culture.

However, mathematics is the most magnificent tool that our species has constructed so far and its power is not even close to being exhausted. We are right to celebrate that fact even if we are not Platonists.