

Superior sums that don't add up to much

How mathematics is taught is in dispute, and our children's knowledge of it is worse than ever.

There is a very curious situation in mathematics at the moment. There are, on the one hand, clear signs of a crisis. There is a strong consensus in the university mathematics world that the mathematical awareness, skills and understanding of pupils completing secondary education have deteriorated rapidly in recent years. Students know less, understand less, have little facility with simple operations, have little idea what is meant by proof, find difficulty in solving any but the shortest of problems.

The clearest evidence for this is that mathematics departments, far from just complaining, have acted. First year courses now include a large amount of revision material, analysis has been pushed into the second year almost everywhere, and the new MSci or MMath is essentially an attempt to reach in four years the standard that was not so long ago attained in three. What is more, this is happening right across the board.

At the same time, however, there appears to be a feeling of satisfaction within the education community. There have been a great many changes in the syllabus, all of them apparently for the better. We are told that the new methods may not cram the same number of facts into the pupils - children are not pots into which information is to be poured - but that they make the students better at problem solving, at thinking for themselves, at understanding mathematics. Academic standards in the schools, we are told, have not declined, they have risen.

Our first year students certainly do know less. They are also not as good at manipulation, the nuts and bolts of elementary mathematics. Unfortunately, they are not better at the other things. On the contrary, they are worse.

It might be argued that the most able students have had to suffer for the benefit of the majority, but we are assured there has been no fall in standards at the top. Besides, the evidence that the middle group are actually doing better is unreliable. The exam results may be better, but that is not the same thing. A-level no longer represents the same level of attainment that it once did: the examination papers now contain more questions which lead the candidate through a number of easy steps and so require less understanding and less synthesis.

It is also clear that GCSE is, as intended, at a lower standard than the O-level it replaced; school teachers report the same difficulty in teaching A-level to pupils with GCSE that we find in teaching university courses to students with present day A-levels. And at an earlier stage, we note that in constructing the National Curriculum Key Stage 3 SAT's, a great deal of care has been taken to ensure that a fixed proportion of 14-year-olds can answer a given question. Far less consideration has been given to the loss of mathematical content in the process of trial and adjustment, or to comparison with an external standard.

The problem is not simply the schools being perhaps too complacent and the universities perhaps too demanding. It goes much deeper than that. There are fundamental disagreements about what mathematics is, what pupils should learn about mathematics at school, what skills they should learn within mathematics both to be used later on in the subject and elsewhere, what else they should be learning while they are learning these and what the priorities are. They go right to the heart of how pupils learn anything.

Central to the issue lies what may be described as the epistemological divide between a traditional emphasis on knowledge as an external landscape, and a post-Piagetian emphasis on knowledge as an individually constructed internal picture. The situation is sometimes regarded as if there were two diametrically opposite positions. On the one hand there are the Absolutists, cultural restorationists who see teaching as presenting a corpus of facts, and learning as a passive process of receiving this. Opposed to them are the Relativists, liberal progressives, for whom teaching means presenting learning activities, and learning is an active process of building a mental network of relations.

But is this a true picture? Are there really two totally incompatible points of view and nothing in between? Does the traditional approach really do nothing more than impart facts? Is there nothing active about the process of internalising knowledge (however acquired) or about solving problems in the traditional way? Can students really construct mental relations without a background of facts to operate with and to relate?

In mathematics teaching, the changes are sometimes seen as a jettisoning of content - Euclidean geometry, the arithmetic of fractions, algebra - to allow time for a careful development of process skills: interpreting, communicating, selecting, applying, and so on. But where an instrumental understanding of content is merely replaced by an instrumental understanding of process, there is no improvement. On the other hand, a relational understanding of content carries with it a relational understanding of process. The converse is not true. Pupils cannot genuinely understand processes if they do not understand the content on which these are supposed to be operating, and if you reduce the latter you inevitably limit the former.

Our concern is not confined to the small proportion of young people who enter university to read mathematics. Many of those who are studying science and engineering are also seriously handicapped by inadequate mathematical backgrounds. But at least we have a chance to repair the damage, even though it is wasteful to cover at university what could have been done at school.

We have no such chance, however, with those we never see. Those who are responsible for school mathematics should remember that the majority of pupils, even those who go on to further or higher education, will do no more mathematics. What is provided for them in the schools is all they are ever going to meet. We are especially concerned that many of them might have responded to real mathematics but find nothing stimulating in the watered down version and so go into other fields instead. Would so many young people be keen to read English at university if they had never encountered real poetry and literature?

We understand perfectly well that schools cannot simply go back to what was done twenty or more years ago. The universities too have had to make major adjustments to the style and content of their mathematics teaching to cope with changed conditions. Outsiders may not be so aware of it, because what we do doesn't usually attract the same sort of attention as what happens in secondary education, but there have been changes and they have been profound. We have not, however, redefined the subject in the process.

How to provide the best possible education in mathematics for all ability ranges and in the world as it is today and with all the problems the schools are having to face is not easy to decide. Nothing can happen, however, until there is an agreement that a problem exists. As long as the educationists persist in telling us that today's students are as well prepared in mathematics as past students with the same grades, when we know that they patently are not, and as long as we look at curriculum documents and see a continuing trend away from what we recognise as mathematics, we are not going to progress.