The Impact of Heterogeneous Trading Rules on the Limit Order Book and Order Flows

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Abstract

In this paper we develop a model of an order-driven market where traders set bids and asks and post market or limit orders according to exogenously fixed rules. The model developed here extends the earlier one of Chiarella and Iori (2002) in several important aspects, in particular agents have heterogenous time horizons and can submit orders of sizes larger than one, determined either by utility maximisation or by a random selection procedure. The model seeks to capture a number of features suggested by recent empirical analysis of limit order data, such as; fat-tailed distribution of limit order placement from current bid/ask; fat-tailed distribution of order execution-time; fat-tailed distribution of orders stored in the order book; long memory in the signs (buy or sell) of trades. We analyze the impact of chartist and fundamentalist strategies on the determination of both the placement level and the placement size, on the shape of the book, the distribution of orders at different prices, and the distribution of their execution time. We compare the results of model simulations with real market data.

1 Introduction

Traders willing to trade in electronic markets can either place market orders, which are immediately executed at the current best listed price, or place limit orders. Limit orders are stored in the exchange's book and executed in the sequence they arrive to the market. A transaction occurs when a market order hits the quote on the opposite side of the market. Transactions are executed using time priority at a given price and price priority across prices.

In the last few years several order driven microscopic models have been introduced to explain the statistical properties of asset prices, we cite in particular Bottazzi et al. (2002), Consiglio et al (2003), Chiarella and Iori (2002), Daniel et. al (2002) Li Calzi and Pellizzari (2003), Luckock (2003) and Raberto et al (2001).

The aim of Chiarella and Iori (2002) was to introduce a simple auction market model to understand how the placement of limit orders contributes to the price formation mechanism. The impact on the market of three different trading strategies: noise traders, fundamentalists and chartists was analyzed. It was shown that the presence of chartists plays a key role in generating realistic looking dynamics, such as volatility clustering, persistent trading volume, positive cross-correlation between volatility and, trading volume and volatility and bid-ask spread.

Recently the empirical analysis of limit order data has revealed a number of intriguing features in the dynamics of placement and execution of limit orders. In particular, Zovko and Farmer (2002) found a fat-tailed distribution of limit order placement from the current bid/ask (with an exponent $\alpha \sim 1.49$). They also find a slow decay of relative limit order autocorrelation (with an exponent $\delta \sim 0.41$). Bouchaud, Mézard and Potters (2002) and Potters and Bouchaud (2003) found a fat-tailed distribution of limit order arrival (with an exponent $\alpha \sim 0.6$, smaller than the value observed by Zovko and Farmer) and a fat-tailed distribution of orders stored in the order book. Challet and Stinchcombe (2001) found a fat-tailed distribution of order execution times (with an exponent $\gamma \sim 1.5$).

The analysis of order book data has also added to the debate on what causes fat tailed-fluctuations in asset prices. Gabaix et al (2003) proposed that large price movements are caused by large order volumes. While a variety of studies have made clear that the mean market impact is an increasing function of the order size, Farmer et al (2004) have shown large price changes in response to large orders are very rare. Furthermore they have shown that an order submission typically results in a large price change when a large gap is present between the best price and the price at the next best quote (see also Weber and Rosenow (2004)). We show in section 3 that in our model large returns are also mostly associated with large gaps in the book.

Lillo and Farmer (2004) also reported, for the LSE, that the sign of orders is a long memory process, with a slowly decaying auto-correlation function (roughly a power-law with a stock dependent exponent $\gamma < 1$). This observation poses the interesting question of how can markets be efficient (price changes reveal very fast decaying temporal correlations) if the sign of future orders would seem to be predictable. The authors suggested that this effect is compensated by long range fluctuations in liquidity and market orders, as revealed by a decrease in the ratio of market order size to the volume at the best price when the order sign is predictable. The same question has been posed by Bouchaud et al (2004a, 2004b), who provide a somewhat different explanation. They suggest that liquidity takers, in order to minimize the impact of their orders on prices, create long-range correlations by splitting their orders into a sequence of small orders, and market efficiency is restored by the market makers who adapt the order flow to mean revert the price and optimize their gains. In this paper we do not enter the debate on how efficiency is re-established but provide a possible explanation for the long memory in order signs as being generated by chartists strategies.

In order to build a model that can incorporate these recent empirical findings of limit order data, we here extend the original model of Chiarella and Iori (2002) in two main respects. First, agents have different time horizons: longer time horizons for fundamentalists and shorter horizons for chartists and noise traders. Second, agents can submit orders of size larger than one by considering two alternative scenarios for the determination of the agents' asset demand function. One scenario merely allows asset demand to be a random function of available wealth. The other considers an asset demand function determined in the traditional economic framework of expected utility maximisation.

We simulate our model and compare its qualitative predictions against the empirically observed properties of prices and order flows in an attempt to provide an explanation for the observed regularities in terms of the different strategies of our population of traders. We show in particular that the introduction of chartist strategies into a population of utility optimizing traders may indeed induce long memory correlations in the order signs and liquidity and a number of other features observed in the data. The paper is structured as follows; in section 2 we model in particular the fundamentalist, chartist and noise trader components of expectations and the way in which agents form their demands for the risky asset. In section 3 we undertake a number of simulations of the model under both forms of asset demand function to determine how well the empirical facts referred to earlier are reproduced. Section 4 concludes

2 The Model

We assume that agents know the fundamental value p^f of the asset, which we take to be constant. Agents also know the past history of prices. At any time t the price is given by the price at which a transaction occurs, if any. If no new transaction occurs, a proxy for the price is given by the average of the quoted ask a_t^q (the lowest ask listed in the book) and the quoted bid b_t^q (the highest bid listed in the book): $p_t = (a_t^q + b_t^q)/2$ (we call this value the mid-point). If no bids or asks are listed in the book a proxy of the price is given by the previous traded or quoted price. Bids, asks and prices need to be positive and investors can submit limit orders at any price on a prespecified grid, defined by the tick size Δ.

The demands of traders are assumed to consist of three components, a fundamentalist component, a chartist component and a noise induced component.

At any time t a trader is chosen, with a given probability λ, to enter the market. The chosen agent, makes an expectation about the spot return, $\hat{r}^i_{t,t+\tau_i}$, that will prevail in the interval $(t, t+\tau_i)$, where τ_i is the agent's time horizon. Agents use a combination of fundamental value and chartist rules to make expectations on stock returns, so that

$$\hat{r}^i_{t,t+\tau} = \frac{1}{g^i_1 + \mid g^i_2 \mid + n^i} \left[g^i_1 \frac{1}{\tau_f} \frac{(p^f - p_t)}{p_t} + g^i_2 \bar{r}_{L^i} + n^i \epsilon_t \right]. \tag{1}$$

where $\bar{r}_{L_i}$ is an average of past returns that is specified below.

The quantities $g^i_1 > 0$ and g^i_2 represent the weights given to the fundamentalist and chartist component respectively. The sign of g^i_2 indicates a trend chasing (> 0) or contrarian (< 0) chartist strategy. Since the degree of fundamentalism and chartism will be spread across agents we model these parameters as random variables independently chosen for each agent with $g^i_1 \sim |N(0, \sigma_1)|$, $g^i_2 \sim N(0, \sigma_2)$. In addition there is a noise induced component to agent's expectations for which we assume $n^i \sim |N(0, \sigma_n)|$ and $\epsilon_t \sim N(0, \sigma_\epsilon)$. The initial term of the right-hand side normalises the impact of the three trading strategies. A trader for whom $g^i_1 = g^i_2 = 0$ is a noise trader. The quantity τ_f is the time scale over which fundamentalists expect price to revert to the fundamental value.

The future price expected at time $t + \tau^i$ by agent i is given by

$$\hat{p}^i_{t+\tau^i} = p_t e^{\hat{r}^i_{t,t+\tau^i} \tau^i}. \tag{2}$$

It is common to assume that the time horizon of an agent depends on its characteristics. Fundamentalist strategies are typically used by long term institutional investors, whilst chartist rules are mainly followed by day traders. Hence we choose the time horizon τ_i of each agent according to[1]

$$\tau^i = \left[\tau \frac{1 + g^i_1}{1 + \mid g^i_2 \mid} \right], \tag{3}$$

where τ is some reference time horizon.

The quantity $\bar{r}^i_L$ is the spot return averaged over the interval L_i, i.e.

$$\bar{r}_{L^i} = \frac{1}{L^i} \sum_{j=1}^{L^i} r_{t-j} = \frac{1}{L^i} \sum_{j=1}^{L^i} \log \frac{p_{t-j}}{p_{t-j-1}}. \tag{4}$$

The time windows L^i are assumed to be proportional to τ^i and here we simply take

$$L^i = \tau^i. \tag{5}$$

[1]The notation $[x]$ refers to the value obtained by rounding up to the next highest integer.

2.1 Risk neutral agents with random demand function

We assume here that agents are risk neutral and only aim to maximize profits in their investment decision. When called to trade, if an agent expects a future price increase (decrease) it decides to buy (sell). We assume that the agent is willing to buy (sell) at a price b_t^i (a_t^i) lower (higher) than its expected future price $\hat{p}_{t+\tau_i}^i$ where the agent calculates its bid and ask according to

$$b_t^i = \hat{p}_{t+\tau^i}^i(1-k^i), \tag{6}$$

$$a_t^i = \hat{p}_{t+\tau^i}^i/(1-k^i), \tag{7}$$

where the k^i are uniformly distributed in the interval $(0, k_{max})$ with $k_{max} \leq 1$. If b_t^i (a_t^i) is smaller (larger) than the current quoted ask (bid) a_t^q (b_t^q) the trader submits a limit order at b_t^i (a_t^i) while if b_t^i (a_t^i) is larger (smaller) or equal to the current quoted ask (bid) the trader submits a market order and trades at the current quote. Prices are discrete and the minimum price tick is Δ.

In this section we assume that agents have a random demand function and that the size of their order is bounded by budget constraints. Agents hold a finite amount of cash C_t^i and stocks S_t^i in their portfolio. The size s^i of the order of agent i is determined as follows:-

If the agent expect a price decrease it sells a random fraction of its assets $s^i = \xi S^i$. If however the agent expects a price increase it invests a random fraction of its cash in the assets according to the rule

$$s^i = \xi C_t^i / b_t^i \text{ if limit order}$$
$$s^i = \xi C_t^i / a_t^q \text{ if market order (agent buys at the current ask).}$$

Here ξ is a random variable uniformly distributed on the interval $(0, 1)$.

Consider the situation in which the traders expect a price increase and decide to buy. If $b_t^i < a_t^q$ the trader submits a limit order at b_t^i. If $b_t^i \geq a_t^i$ the trader submits a market order at a_t^q. If the demand available on the book at the ask is sufficiently large, the agent buys all the stocks at the ask price; otherwise, it takes the available supply and then moves on to check the second best ask price, continuing the process until the agent has no more stocks to buy or there are no more sell orders in the book at a price smaller than b_t^i. If the latter is the case, the agent places a limit order at b_t^i for the quantity of stock it still wishes to buy. If the limit order is still unmatched at time $t + \tau^i$ it is removed from the book.

2.2 Risk-averse agents

In this section we consider the case of risk averse agents. The optimal composition of the agent's portfolio is determined by trading-off expected return against expected risk. The number of stocks an agent is willing to hold in its portfolio at a given price level p depends on the choice of the utility function [2]. With an exponential utility of wealth

[2]Equation 8 can be derived on the basis of mean-variance one-period portfolio optimization

function [3] $U(W) = -e^{-\alpha W}$ the optimal composition of the portfolio, i.e. the number of stocks the agent wishes to hold, is given by

$$\pi^i(p) = \frac{\log(\hat{p}^i_{t+\tau^i}) - \log(p)}{\alpha^i \sigma_t p}, \tag{8}$$

which is independent of wealth.

Here σ_t is the variance of returns, which is assumed to be calculated in the same way by all the agents[4] and is estimated as

$$\sigma_t = \frac{1}{N} \sum_{j=1}^{N} [r_{t-j} - \bar{r}_N]^2, \tag{9}$$

where

$$\bar{r}_N = \frac{1}{N} \sum_{j=1}^{N} \log \frac{p_{t-j}}{p_{t-j-1}}. \tag{10}$$

The coefficient α^i measures the risk aversion of agent i. We assume that fundamentalists are more risk averse than noise traders and chartists, which would be captured by

$$\alpha^i = \alpha \frac{1 + g_1^i}{1 + \mid g_2^i \mid},$$

where α is some reference level of risk aversion.

If the amount $\pi^i(p)$ is larger than the number of stocks already in their portfolio then agents decide to buy, if smaller they decide to sell.

Following Bottazzi et al (2003) we first estimate the price level p^* at which agents are satisfied with the composition of their current portfolio,

$$\pi(p^*) = \frac{\log(\hat{p}^i_{t+\tau_i}) - \log(p^*)}{\alpha^i \sigma_t} = S_t^i. \tag{11}$$

This equation also admits a unique solution with $0 < p^* < \hat{p}^i_{t+\tau_i}$. Agents are willing to buy at any price $p < p^*$ and willing to sell at any price $p > p^*$. Note that agents may wish to sell even if they expect a future price increase.

Nonetheless as we want to impose budget constraints we need to restrict ourselves to values of $p \leq \hat{p}^i_{t+\tau_i} = p_M$ to ensure $\pi(p) > 0$ and rule out short selling. Furthermore to assure that an agent has sufficient cash to purchase the desired stocks, the smallest value of p we can allow is defined by the condition

$$\pi(p_m) - S_t^i = \frac{C_t^i}{p_m}.$$

Again one can easily show that this equation also admits a unique solution.

[3] We recall that this utility function belongs to the CARA (constant absolute risk aversion) class.

[4] We could alternatively have assumed that each agent estimates the variance over a different time window (for example L^i)

The possible values at which an agent can satisfactorily trade are in the interval (p_m, p_M). Agents randomly pick a price p in the interval (p_m, p_M) and if $p < p^*$ they submit a limit order to buy an amount

$$s^i = \pi^i(p) - S^i_t,$$

while if $p > p^*$ they submit a limit order to sell an amount

$$s^i = S^i - \pi^i(p).$$

If $p < p^*$ and $p > a^q_t$ the buying order can be executed immediately at the ask. An agent in this case would submit a market order to buy an amount

$$s^i = \pi^i(a^q_t) - S^i_t,$$

while if $p > p^*$ and $p < b^q_t$ the agent would submit a market order to sell an amount

$$s^i = S^i - \pi^i(b^q_t).$$

If the depth at the bid (ask) is not enough to fully satisfy the order, the remaining volume is executed against limit orders in the book at quotes above (below) p. If there are not enough orders in the book to execute the incoming market order fully the remaining volume is converted into a limit order at price p.

3 Simulation Analysis of the Book

In this section we analyze, via numerical simulations, various properties of the book. In the simulations we considered (in succession) three kinds of trading rules, noise traders only (black), fundamentalists and noise traders only (red), and, fundamentalists, chartists and noise traders (green). We ran the simulation with parameters $\sigma_1 = 0$, $\sigma_2 = 0$ (noise traders only), $\sigma_1 = 0$, $\sigma_2 = 3$ (fundamentalists and noise traders only) and $\sigma_1 = 3$, $\sigma_2 = 4.25$ (fundamentalists, chartists and noise traders), for the risk neutral agents model where demand is randomly chosen. For the risk averse agents model we chose $\sigma_1 = 3$, $\sigma_2 = 2$ (fundamentalists, chartists and noise traders). For both models $\tau = 200$, $\Delta = 0.01$, $\lambda = 0.5$, $\sigma_n = 1$. The other parameter for the risk neutral agents model were $k_{max} = 0.5$, and for the risk-averse agents model $\alpha = 0.1$. Agents are initially assigned a random amount of stock uniformly distributed in the interval $[0, 100]$ and an amount of cash $C_0\,[0, 100]$. The initial stock price is chosen at $p_0 = 1000$, the fundamental price level is also set so that $p_f = 1000$, and the $C_0 = p_0$.

The results reported here are the outcome of a long simulation of the model. To test the robustness of the results we have repeated the simulation with a number of different random seeds and also varying the parameter set within a small neighborhood of those used here. We have found that the qualitative features reported below are fairly robust to these variations.

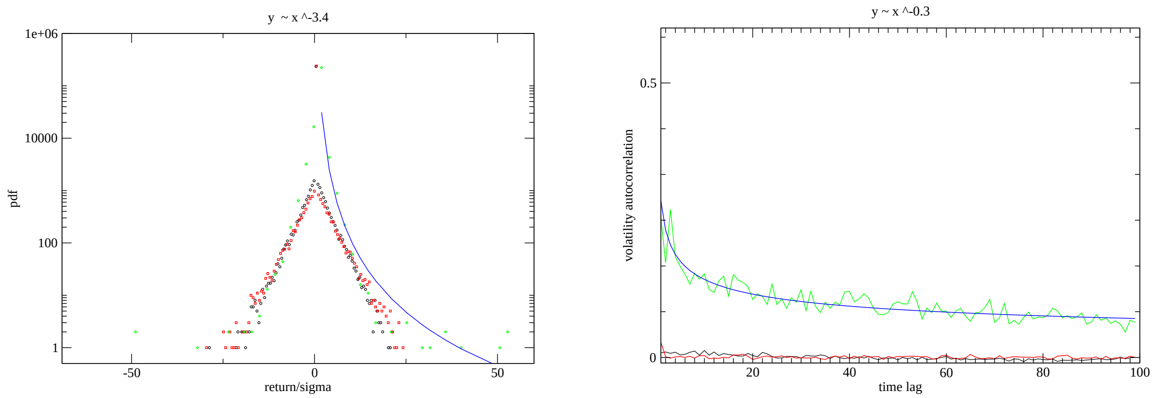

Figure 1: Distribution of normalized returns (right) and autocorrelations of absolute returns (left). The random demand case.

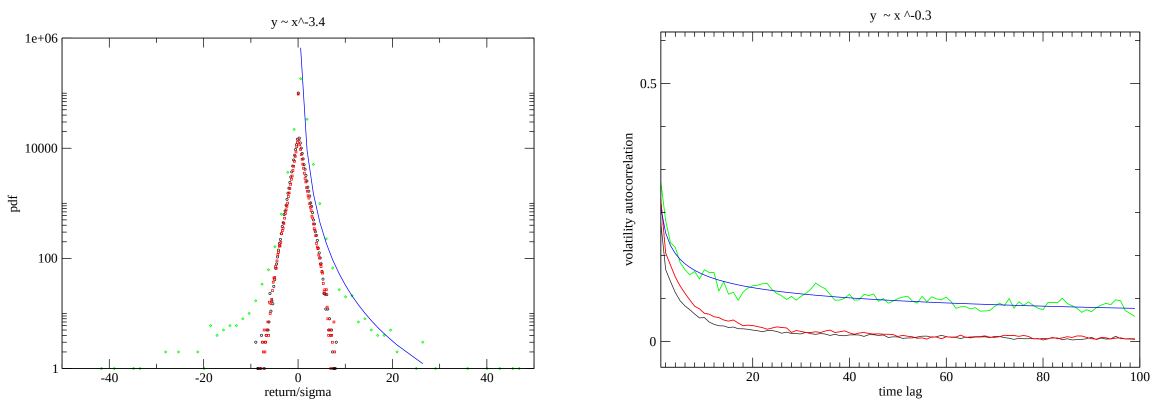

Figure 2: Distribution of normalized returns (left) and autocorrelations of absolute returns (right). The utility based demand case.

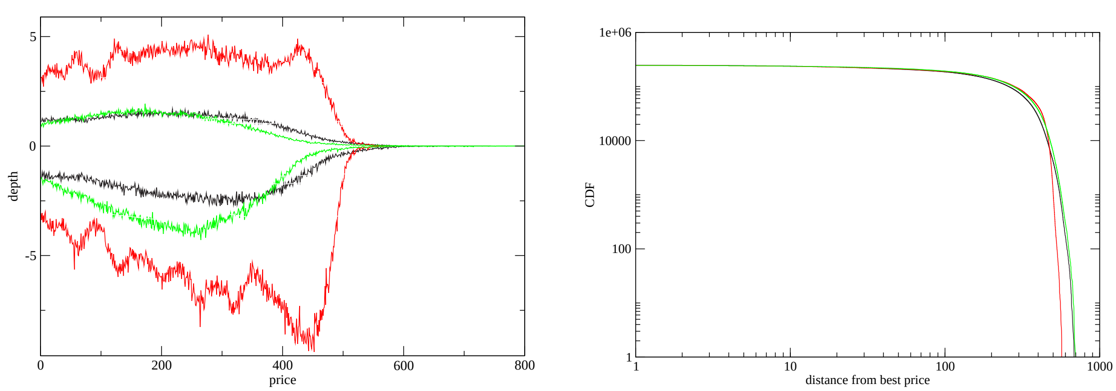

Figure 3: The book shape (left) and distribution of order placement from the best price (right). The random demand case.

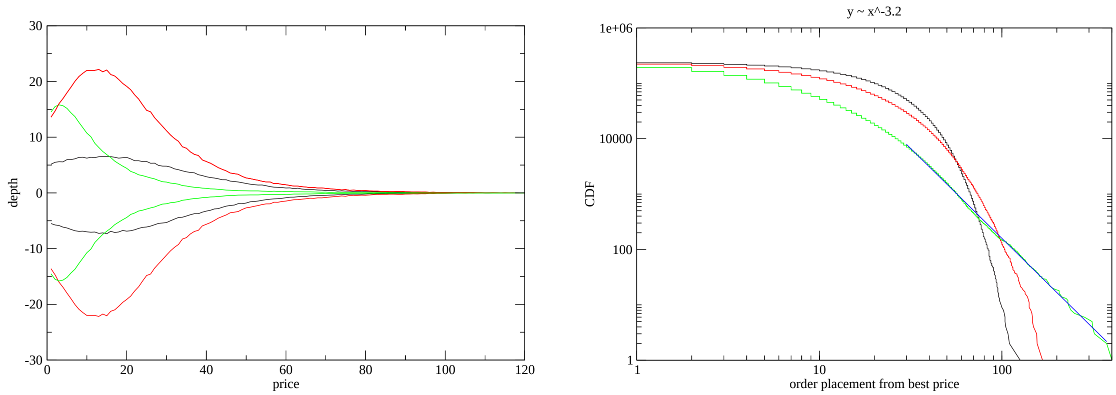

Figure 4: The book shape (left) and cumulative distribution of orders from the best price (right). The utility based demand case.

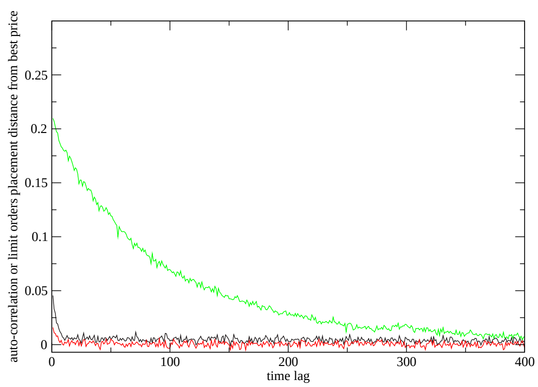

Figure 5: Autocorrelation of order placement distance from best price. The utility based demand case.

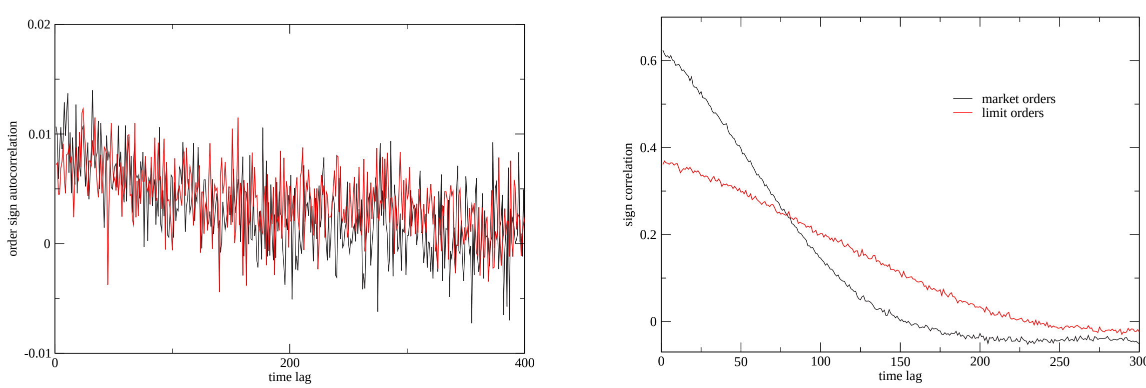

Figure 6: Autocorrelation of limit (black) and market (red) order signs with contrarians (left) and without contrarians (right). The utility based demand case.

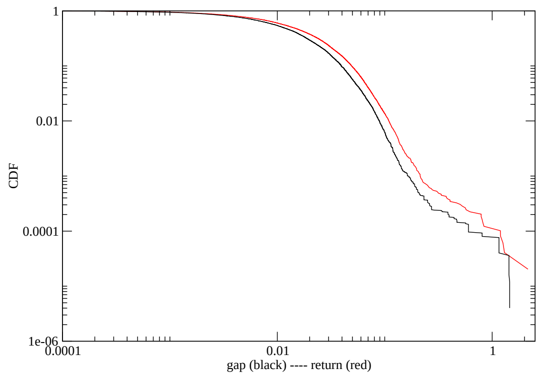

Figure 7: The cumulative distribution of the size of the first gap matches the distribution of returns. The random demand case.

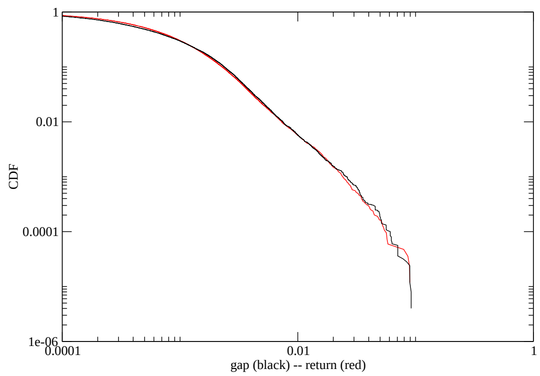

Figure 8: The cumulative distribution of the size of the first gap matches the distribution of returns. The utility based demand case.

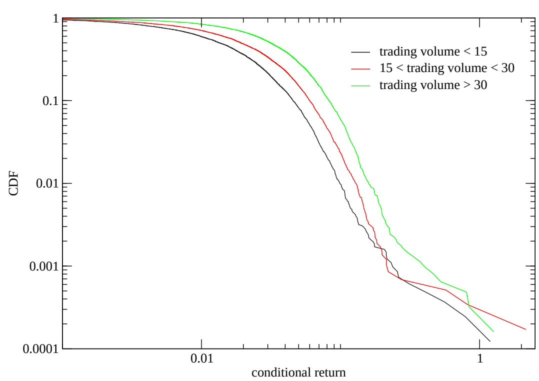

Figure 9: The cumulative distribution of returns conditional on order size. The random demand case.

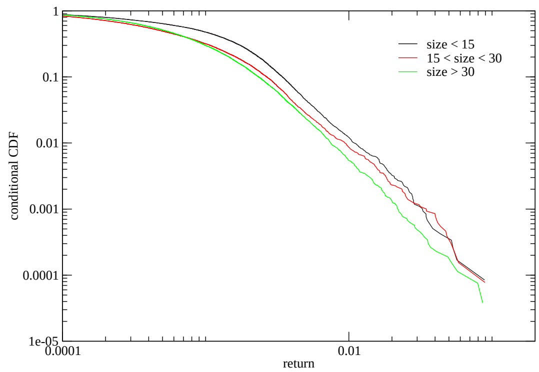

Figure 10: The cumulative distribution of returns conditional on order size. The utility based demand case.

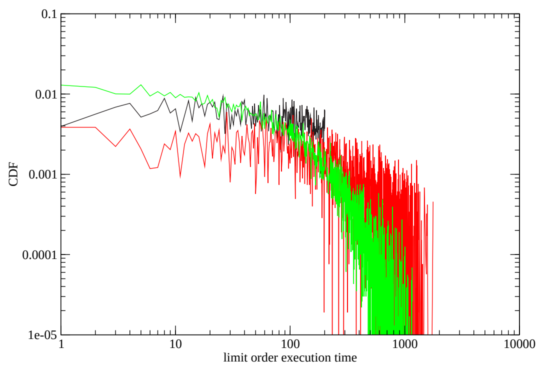

Figure 11: The cumulative distribution of limit order execution time. The random demand case.

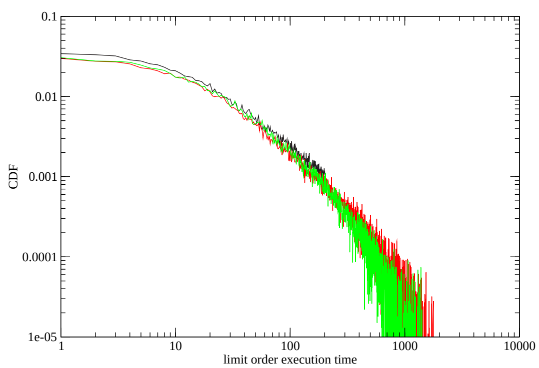

Figure 12: The cumulative distribution of limit order execution time. The utility based demand case.

Figure 1 (left) displays the distribution of normalized returns with noise traders, fundamentalists and chartists when demand is randomly determined. The distribution decays exponentially when chartists are absent, and hyperbolically with an exponent $\mu = 3.4$ when chartists are introduced into the market. In both models we find no correlation for returns. Autocorrelations of absolute returns with noise traders, fundamentalists and chartists are displayed on the right. The autocorrelation has a power law decay (continuous black line) $C(\delta) \sim \delta^{-\beta}$ with an exponent $\beta = 0.30$ when chartists are introduced into the market in the case of the random demand.

Figure 2 displays the same quantities in the case of demand functions derived from utility maximization. Here also, when chartists rules are activated, the distribution decays hyperbolically with an exponent $\mu = 3.4$. Similarly the autocorrelation continues to have a power law decay with an exponent $\beta \sim 0.3$ (the exponent is larger than one in the other two cases without chartists, indicating only short memory). We also see from the graph on the left in figure 2 that the utility based model more readily yields the fat tail behaviour observed in actual returns.

Figure 3 (left) displays the book shape in the case of random demand. Prices are relative to the midpoint. Clearly the book is not symmetric, and when fundamentalists dominate the shape becomes very irregular. Also when fundamentalists dominate the book appears more full. The main reason for this effect is that the cancellation rate is lower for fundamentalist orders, given that their time horizon is longer (see equation 3). The distribution of order placement from the best price (right) decays abruptly, showing no power-law decay. In the case of utility based demand functions, see Figure 4, the book has a far more realistic shape. Here, chartist strategies generate a longer, power law, tail in the distributions of orders in the book in qualitative agreement with empirical finding (Bouchaud at el. (2004)).

In Figure 5 we see that in the model with utility based demand chartist strategies generate positive autocorrelation of limit order placement, as empirically detected by Zovko and Farmer (2002). We did not detect any significant correlation in the random demand model and hence do not display the graph for this situation.

Next we consider the autocorrelation of order signs in the utility based demand model in Figure 6. Note that we found no autocorrelation without chartists present in both models and no correlation in the random demand case even with chartists. In Figure 6 (left) we find very low autocorrelation functions both for market and limit orders when chartists are present. The reason why the sign autocorrelation is so low in both models is because of the the large fraction (about 50%) of contrarian rules. On the right side of Figure 6, we plot the utility based demand case case without contrarians. As expected signs exhibit now much higher and slower decaying correlations.

Next we consider the cumulative distribution of the size of the first gap (see Figures 7 and 8 (black line)). Under both types of demand functions these become fat tailed when chartists are present in the market (the only case shown here). Furthermore, as shown in the same figures, the cumulative distribution of returns (red lines) closely follows the distribution of the size of the first gap, indicating, as suggested by Farmer and Lillo (2004), that the presence of large gaps at the best price is what drives large price changes. The reason why chartist strategies generate large gaps in the book, as well as why they generate power law distributions of orders from the best price, is to be found in the expectation formation mechanism. Chartist rules lead to higher (positive and negative) expected price changes and consequently wider price intervals over which orders can be placed.

To investigate the issue further we also compute the distribution of returns conditional on the size of incoming market orders. To this end, we split orders into three groups with approximately the same amount of orders. The first group includes orders for buying or selling less than 15 stocks, the second group orders of size between 15 and 30 stocks and the last group orders of size larger than 30 stocks. Figures 9 and 10 display the cumulative return distribution for the three groups and show that all three curves are fat tailed with the tails having approximately the same slope.

Figures 11 and 12 display the cumulative distribution of limit order execution time. Even if a power law decay is not possible because of the natural cut-off given the distribution of time horizons, the utility based demand model seems to have a better agreement with empirical findings than the random demand model.

4 Conclusion

We have extended the earlier model of Chiarella and Iori (2002) on double auction markets in a number of ways. In particular we have allowed the demand for stock by agents to fluctuate and agents also may have heterogenous investment horizons. We first consider the case of risk-neutral agents with random demand functions and then risk averse agents forming demands by maximising one-period expected CARA utility. We capture many of the dimensions of agent heterogenity by drawing agent characteristics from statistical distributions.

We find that the version of the model with risk-averse agents captures fairly well a number of recently observed stylized facts of double auction markets, such as fat-tailed distribution of limit orders placement from current bid/ask; fat-tailed distribution of order execution-time; fat-tailed distribution of orders stored in the order book.

We also find that the inclusion of a chartist element into agent expectations seems to be necessary to generate realistic return patterns as well as the stylised facts just referred to. Chartist strategies may also introduce long memory in the sign of order submission. This effect has been observed in real data and has been attributed to order splitting strategies. Here we show that, while order splitting may be one of the mechanisms that enerates a slowly decaying, positive correlation of order signs in real markets, chartist strategies may also contribute and partly explain this effect.

Our paper also contributes to the debate on what generates large price changes in stock markets. Our simulations seem to confirm the picture proposed by Farmer and Lillo, that large returns are driven by large gaps in occurring in price level adjacent to the best bid and best ask.

Future research could aim to enrich the economic framework of the model. For instance in place of allowing the weights on the fundamentalist, chartist and noise trader components in (1) to the randomly selected they could be chosen on the basis of some fitness measure as in Brock and Hommes (1998). A further question of interest would be to see how the order book and order flow are affected if we assume agents have CRRA (constant relative risk aversion) utility functions, in which case their asset demands depend on their level of wealth. We know from Chiarella, Dieci and Gardini (2004) that in this type of modelling framework CRRA and CARA utility functions lead to different types of dynamics.

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