1. Let $P_1, ..., P_m$ be m different points in the (x, y) plane. The for each i there is a polynomial $H_i = H_i(x, y)$ such that $H_i(P_j) = \delta_i^j$.

2. Suppose we have a conic C on the plane and a point p on it. Suppose we can draw a line trough any two points on the plane. Using Pascal's theorem guess how to draw the tangent line to C at p.

3. Consider the subset of $\mathbb{R}^2$ given by one half of the hyperbola $x^2 - y^2 = 1$. Can this subset be given by system of polynomial equations? Hint. Use Bézout theorem.

4. Prove that each hexagon such that three intersection points of lines containing pairs of hexagon's opposite sides lie on a line is inscribed in a conic.

5. Give examples of polynomials $F, G \in K[x, y]$ such that:

A) $\#(F = 0 \cap G = 0) < dim(K[x, y]/(F, G))$.

B) $dim(K[x, y]/(F, G)) < deg(F) \cdot deg(G)$.

(here (F, G) denotes the ideal generated by F and G.)

6. Find the dimension of the space of polynomials in $K[x_1, ..., x_n]$ of degree at most d.

7. Prove that any reducible homogenous polynomial can be represented as a product of homogeneous polynomials of positive degree.

8. Decompose over $\mathbb{C}$, $\mathbb{R}$, and $\mathbb{F}_2$ the following polynomials in the product of irreducible polynomials: 1) $x^4 + y^4$. 2) $x^2 + y^2 + z^2$.

9. Let $F = \sum a_i z_0^i z_1^{d-i}$ be a homogeneous polynomial in two variables, $a_i \in \mathbb{C}$. Prove that it can be written as $F = \Pi_{k=1}^d (b_k z_0 + c_k z_1)$.

10. Find points at infinity of the following two curves (these curves are in $\mathbb{C}^2$ and you need to find their intersection with line at infinity in $\mathbb{P}^2_{\mathbb{C}}$).

$x_1^2 + x_1x_2 - 2x_2^2 + x_1 - 5x_2 + 7 = 0$, $x_1^3 + x_1^2x_2 - 3x_1x_2^2 - 3x_2^3 + 2x_1^2 - x_1 + 5 = 0$.

11. Let let $A_1, ..., A_5$ be distinct points on the plane. Prove:

1) There is a unique conic that contains points A_i iff no four of A_i lay on one line.

2) If no three of A_i lay on one line, then the conic is irreducible.

12. Give examples of pairs of conics in $\mathbb{P}^2_{\mathbb{C}}$ that intersect in exactly 4 points; 2 points; 3 points; 1 point. Specify what are the multiplicities of the intersections of the conics.

13. Let K be a finite field and $P \in K[x_1, ..., x_n]$. Suppose that the degree of P less than $|K|$ and it vanishes at each point of K^n. Prove that P is identically zero.