Full Name: ____________________
(block capitals)

Student Number: ____________________ Tutorial Group: __________

4CCM115A Numbers and Functions: Test 4

CALCULATORS MAY NOT BE USED

ANSWER GRID: put a cross in ONE BOX for the correct answer for each question. If you change your mind and want to correct your answer, obliterate your incorrect answer by shading its box, and put a new cross in the box for the correct answer.

	a	b	c	d	e
1					
2					
3					
4					

MARKS: each correct answer = +5, incorrect = −1, none (or more than one) = 0.

Do any rough working on the back of this sheet, or on a NAMED separate sheet.

1. Which one of the following statements holds true?

(a) The sequence $s_n = 5^n/n!$ is increasing
(b) The sequence $s_n = \dfrac{2^n+1}{2^n+3}$ is decreasing
(c) The sequence $s_n = |n^2 - 100|$ is decreasing
(d) The sequence $s_n = \sum_{k=1}^{n} \frac{1}{k^3}$ is increasing
(e) None of the above statements is true

2. Let $s_n = \dfrac{n}{2n^2+1}\sin(\pi n/2)$, $n \in \mathbb{N}$. Which one of the following statements holds true?

(a) s_n has no limit points
(b) s_n has exactly one limit point
(c) s_n has exactly two limit points
(d) s_n has exactly three limit points
(e) None of the above statements is true

3. Let $s_n = (1+3^{-n})^{3^{n+1}}$. Which one of the following statements holds true?

(a) $s_n \to 1$ as $n \to \infty$
(b) $s_n \to e$ as $n \to \infty$
(c) $s_n \to e^3$ as $n \to \infty$
(d) $s_n \to e^{-3}$ as $n \to \infty$
(e) None of the above statements is true

4. Which one of the following statements holds true?

(a) Any Cauchy sequence has a limit point.
(b) Any monotone sequence has a limit point.
(c) If a sequence is monotone then it is a Cauchy sequence.
(d) If a sequence is Cauchy then it is monotone.
(e) None of the above statements is true.

End of Test

Solutions

	a	b	c	d	e
1				×	
2		×			
3			×		
4	×				

1. The correct answer is (d). Indeed, in this case we have $s_{n+1} - s_n = 1/(n+1)^3 \geq 0$, and so s_n is increasing.

For the other sequences, we have:

(a) the sequence is neither increasing nor decreasing. It is decreasing for all large values of n, $n \geq 5$.

(b) $s_n = 1 - \frac{2}{2^n+3}$. The sequence $\frac{2}{2^n+3}$ is decreasing, so s_n is increasing;

(c) the sequence is neither increasing nor decreasing. It is increasing for all large values of n, $n \geq 10$.

2. The correct answer is (b). Indeed, it is easy to see that the sequence converges to zero and so has a unique limit point, which is the limit of this sequence.

3. The correct answer is (c). Indeed, let $t_n = \left(1 + \dfrac{1}{n}\right)^n$. We know that $t_n \to e$ as $n \to \infty$. Thus, any subsequence of t_n also converges to e. In particular, $t_{3^n} \to e$ as $n \to \infty$. Next, $s_n = (1 + 3^{-n})^{3^n \cdot 3} = \left((1 + 3^{-n})^{3^n}\right)^3 = (t_{3^n})^3$, and so by the algebra of limits, $s_n \to e^3$.

4. The correct answer is (a). Indeed, the Cauchy convergence criterion states that if a sequence is Cauchy, it converges. Since it converges, it has a unique limit point which coincides with its limit. For the other statements, we have:

(b) is false; counterexample: $s_n = n$;

(c) is false; counterexample: $s_n = n$;

(d) is false; counterexample: $s_n = (-1)^n/n$.