Solutions

	a	b	c	d	e
1	×				
2			×		
3		×			
4				×	

1. The correct answer is (a). Indeed, we have $s_n = \frac{n^2}{n^2+1} = 1 - \frac{1}{n^2+1}$. The sequence $n^2 + 1$ is obviously increasing, and so $\frac{1}{n^2+1}$ is decreasing, and so $-\frac{1}{n^2+1}$ is increasing. It follows that s_n is increasing.

For the other sequences, we have:

(b) is not monotone,

(c) is decreasing,

(d) is not monotone.

2. The correct answer is (c). Indeed, the sequence takes values $a, a, 1, a, a, 1, a, a, 1, \ldots$, where $a = \cos(2\pi/3) = \cos(4\pi/3)$. Thus, it is clear that the sequence has exactly two limit points: a and 1.

3. The correct answer is (b). Indeed, let $t_n = \left(1 + \frac{1}{n}\right)^n$. We know that $t_n \to e$ as $n \to \infty$. Thus, any subsequence of t_n also converges to e. In particular, $t_{2n} \to e$ as $n \to \infty$. Thus, we have

$$s_n = t_{2n} \left(1 + \frac{1}{2n}\right)^2 \to e \times 1^2 = e$$

as $n \to \infty$.

4. The correct answer is (b); this was one of the theorems proved in the lectures.

(a) is false; counterexample: $s_n = 1/n$.
(c) is false; counterexample: $s_n = (-1)^n/n$.
(d) is false; counterexample: $s_n = (-1)^n/n$.