Solutions

	a	b	c	d	e
1			×		
2					×
3			×		
4				×	

1. The correct answer is (c). Indeed, we have

$$\lim_{n\to\infty} \frac{2^{\frac{n}{2}} + n^2 2^n + n 2^{3n+\frac{3}{n}}}{n 8^{n+1} + n^2 8^{\frac{n}{2}+\frac{2}{n}}} = \frac{(\sqrt{2})^n + n^2 2^n + n 8^n 8^{1/n}}{n 8 \cdot 8^n + n^2 (2\sqrt{2})^n 16^{1/n}} = (*).$$

Dividing by $n8^n$, we get:

$$(*) = \frac{n^{-1}(\sqrt{2}/8)^n + n(1/4)^n + 8^{1/n}}{8 + n(\sqrt{2}/4)^n 16^{1/n}} \to \frac{0+0+1}{8+0} = \frac{1}{8}.$$

2. The correct answer is (e). Indeed:

$$\frac{n2^n}{2^{n/2}} = n2^{n/2} \to \infty \text{ as } n \to \infty;$$
$$\frac{n^2 + n^{-2}}{n + n^{-1}} = n\frac{1 + n^{-4}}{1 + n^{-2}} \to \infty;$$
$$n! 2^{-n} \to \infty \text{ as } n \to \infty;$$
$$5^{-\frac{1}{n}} 5^n \to \infty \text{ as } n \to \infty,$$

and so all of the statements (a)–(d) are incorrect.

3. The correct answer is (c). Indeed, using Theorem 5.6 from the lecture notes, we have:

(a) $3^n - n^2 3^{n/2} = 3^n(1 - n^2 3^{-n/2}) \to \infty$ as $n \to \infty$;

(b) $e^{3n} - 3n^3 e^{n/3} = e^{3n}(1 - 3n^3 e^{-8n/3}) \to \infty$ as $n \to \infty$;

(c) $\sqrt[3]{n} - 3\sqrt{n} = -3\sqrt{n}(1 - \frac{1}{3}n^{-1/6}) \to -\infty$ as $n \to \infty$;

(d) $3^{1/n} - 3^{-1/n} = 3^{1/n} - (1/3)^{1/n} \to 1 - 1 = 0$ as $n \to \infty$.

4. The correct answer is (d). Indeed, we have:

(a) If $s_n = 1/n$, $t_n = n$, then $s_n t_n = 1$. Thus, (a) is wrong.

(b) The same example shows that (b) is wrong.

(c) The same example shows that (c) is wrong.

(d) is Theorem 5.3.