

CM115A, CM115B Numbers and Functions: Assignment 1

Solutions to this assignment must be handed in at the **start** of the tutorial you attend during week 3 (week starting 10 October 2011). Assignments handed in late will not normally be marked. It is essential that you write your NAME, STUDENT NUMBER and GROUP NUMBER on your work.

You should attempt all questions. Some questions may require you to think hard. You will probably not immediately be able to see how to proceed. Keep trying!

The amount of time spent on study for this course outside the scheduled lecture and tutorial hours (including time taken completing the assignment and time taken going through the notes after each lecture and before the next) will vary from student to student, but it should not normally be less than 4 hours per week.

1. List all elements of the set $\{x \in \mathbb{N} \mid -3 < x \leq 5\}$. **Answer:** 1, 2, 3, 4, 5
2. Let K be the set of all students at KCL, M be the set of all male students at KCL, and N be the set of all students who take the Numbers and Functions module. Describe in words the following sets:

$$K \setminus M, \quad M \cap N, \quad N \setminus M, \quad M \setminus N, \quad K \setminus (M \cup N).$$

Answer: $K \setminus M$ = all female students at KCL; $M \cap N$ = all male students who take Numbers and Functions; $N \setminus M$ = all female students who take Numbers and Functions; $M \setminus N$ = all male students who do not take Numbers and Functions; $K \setminus (M \cup N)$ = all female students who do not take Numbers and Functions.

3. Give a more direct description of the following sets:
 - (a) $(-1, 1) \cap [-1, 2)$; **Answer:** $(-1, 1)$
 - (b) $(-\infty, 0) \cap (0, \infty)$; **Answer:** \emptyset
 - (c) $(-\infty, 0] \cap [0, \infty)$; **Answer:** $\{0\}$
 - (d) $[1, \infty) \setminus (1, 2)$; **Answer:** $\{1\} \cup [2, \infty)$
 - (e) $[-1, 1] \setminus (-1, 1)$; **Answer:** $\{-1, 1\}$
 - (f) $(-\infty, 0) \cup (-1, \infty)$; **Answer:** \mathbb{R}
 - (g) $(-\infty, 1) \cap (-3, \infty) \cap [0, 5]$. **Answer:** $[0, 1)$
4. Mark each of the following statements true or false:
 - (a) For any sets A and B , if $A \subset B$ and $B \subset A$ then $A = B$; **Answer:** true
 - (b) For any sets A and B , if $A \subset B$ then $A \neq B$; **Answer:** false
 - (c) For any sets A and B , one has $A \cap B \subset A \cup B$; **Answer:** true

- (d) There exist sets A and B such that $A \setminus B = B \setminus A$; **Answer:** true
- (e) For all sets A and B , one has $A \setminus B = B \setminus A$; **Answer:** false
- (f) For any set A , there exists a set B such that $A \cup B = A \cap B$. **Answer:** true
- (g) For any finite sets A and B , one has $\#(A \cup B) = (\#A) + (\#B)$. **Answer:** false
- (h) For any finite sets A and B , one has $\#(A \cap B) \leq \#A$. **Answer:** true
5. Let A be any set. Describe the sets $A \cup A$, $A \cap A$, and $A \setminus A$. **Answer:** $A \cup A = A$, $A \cap A = A$, and $A \setminus A = \emptyset$.
6. Give a more direct description of the following sets:
- (a) $A = \{x \in \mathbb{R} \mid x^2 - 8x + 15 = 0\}$ **Answer:** $A = \{3, 5\}$
- (b) $B = \{x \in \mathbb{N} \mid -11 < x \leq -7\}$ **Answer:** $B = \emptyset$
- (c) $C = \{x \in \mathbb{Z} \mid -11 < x \leq -7\}$ **Answer:** $C = \{-10, -9, -8, -7\}$
7. Give a more direct description of the following sets:
- (a) $\bigcap_{j=1}^{\infty} [j, \infty)$ **Answer:** \emptyset
- (b) $\bigcup_{j=1}^{\infty} [j, \infty)$ **Answer:** $[1, \infty)$
- (c) $\bigcup_{j=1}^{\infty} (0, 1/j)$ **Answer:** $(0, 1)$
- (d) $\bigcup_{n=1}^{\infty} [1 + \frac{1}{n}, 1 + n]$ **Answer:** $(1, \infty)$
- (e) $\bigcap_{n=1}^{\infty} [1 + \frac{1}{n}, 1 + n]$ **Answer:** $\{2\}$