Dynamics of a Spinning Disk Impacting with Friction

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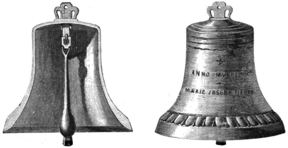
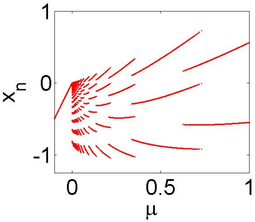
Piecewise smooth dynamical systems (PWS DS)

PWS systems are dynamical systems for which orbits lose smoothness as they intersect certain manifolds.

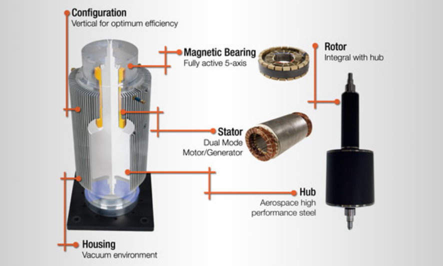




- harbour rich and fascinating dynamics, e.g. period-adding & grazing.
- can model impacts and / or friction
- arise in many applications:
  - firing neurons model [Bressloff et al., 1990]
  - switching phenomena in electrical circuits [di Bernado et al., 1998]
  - church bells [Hinrichs, Oestereich & Popp, 1998]
  - earthquakes [Virgin, 2012]
  - problems with noise [Simpson]
- have rich mathematical theory: B., Hogan, Kunze, Küpper, Nordmark

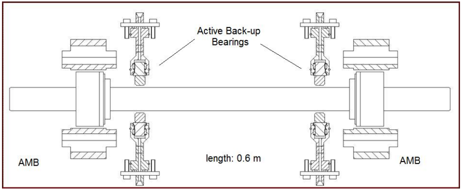
Applications:

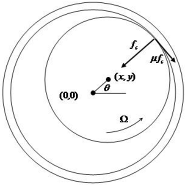
- Turbomolecular Pumps
- Turbines (e.g. Gas)
- Flywheel Energy Storage (VYCON)
- Cutting spindles

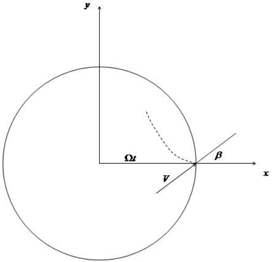


The Magnetic Bearing System

Dynamics is a combination of free rotor motion interrupted by impacts.







Assumption: Ω remains constant even at impact [Keogh & Cole, 2003]

The rotor in free motion in Cartesian coordinates (x, y):

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = e\Omega^2 \cos(\Omega t + \phi) \quad (1)$$

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = e\Omega^2 \sin(\Omega t + \phi) \quad (2)$$

if $r(t) := \sqrt{x(t)^2 + y(t)^2} < c_R$.
Note: ξ = damping ratio, ω_n = undamped frequency, e = unbalance eccentricity, ϕ = unbalance phase, Ω = const. rotational speed.

A instantaneous contact occurs when $r(t) = c_R$. Then reset law in polar coordinates is:

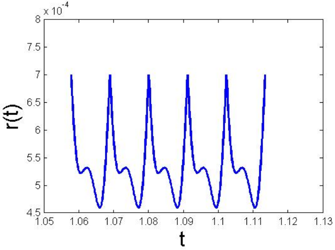
$$\dot{r}^+ = -d\dot{r}^- \quad (3)$$

$$\dot{\theta}^+ = \dot{\theta}^- - (1 + d)\mu\dot{r}^-/c_R. \quad (4)$$

Note: d = coeff. of restitution, μ = coeff. of friction, c_R = radial clearance.
[Keogh & Cole, 2003]

Stable Periodic Impacts

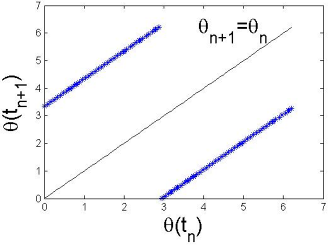
Impact map $P : (t_n, \theta_n, r'^{-}_n, \theta'^{-}_n) \to (t_{n+1}, \theta_{n+1}, r'^{-}_{n+1}, \theta'^{-}_{n+1})$ yields periodic, quasi-periodic and chaotic orbits.



For example:

- stable: $\max |\lambda_i| \leq 1$
- contracting
- $\dot{r}^-$, $\dot{\theta}^-$ same value at each impact

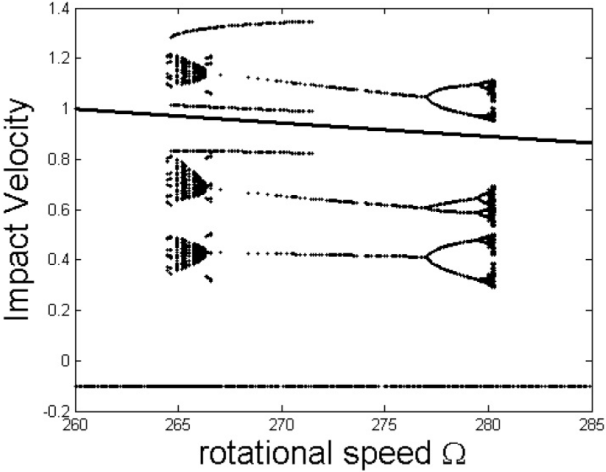
where $\lambda_i =$ eigenvalues of P, for $i = 1, 2, 3, 4$.

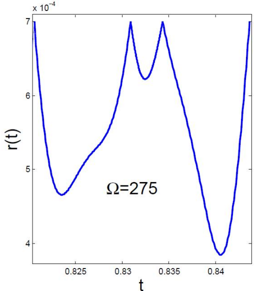


Stable Manifolds: Smooth Bifurcations

Varying rotational speed Ω while fixing all other parameters yields stable impacting orbits via Monte Carlo method.

- Period doubling & Hopf bifurcation
- boundary crises

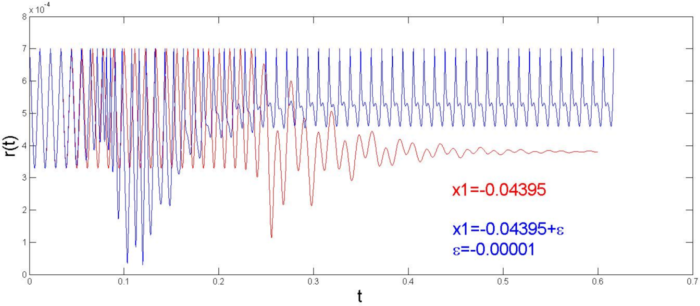




Grazing Manifold

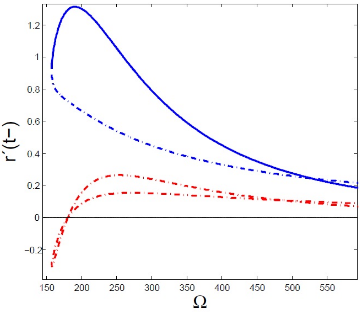
Grazing occurs when radial impact velocity, $r'(t-)$, is zero. Perturbing initial radial velocity gives

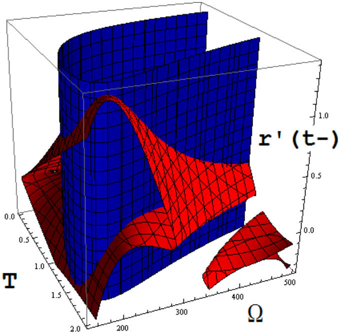
- non-impacting orbit for $\epsilon \geq 0$
- transient giving rise to stable p.o. for $\epsilon < 0$



Fixed point continuation

- 4 solutions are intersection of two algebraic surfaces
- coincident folds
- one solution is virtual but becomes admissible at grazing bifurcation, $r'(t-) = 0$.





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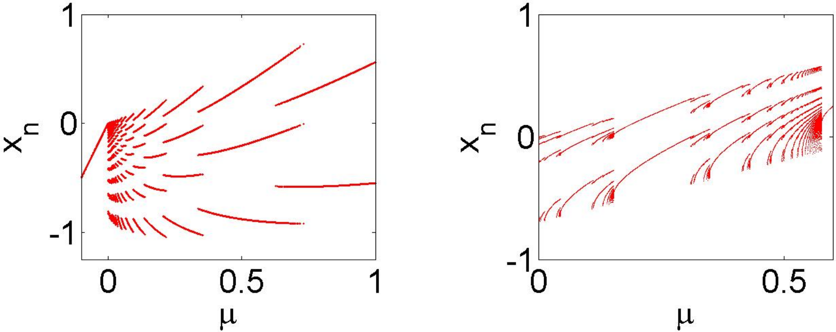
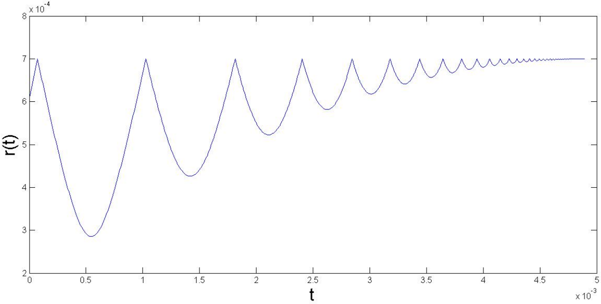
Grazing Bifurcations can give rise to Period Adding cascades or Farey tree sequences in $1D$ maps



Chattering Phenomenon

Chattering: large (∞) number of impacts in finite time.



Open question: what comes after chattering? Experimentally observed

- rolling: forward or backward whirl
- sliding: forward rub

$\Longrightarrow$ Require different reset law.

Conclusions

We have observed smooth

- Period doubling, fold and Hopf bifurcations
- boundary crisis

... and non-smooth dynamics

- grazing bifurcation
- chattering

Open questions

- Global and local existence theory
- Require different reset law to model sliding and rolling
- Material damage analysis
- Compare to experimental data (noise).

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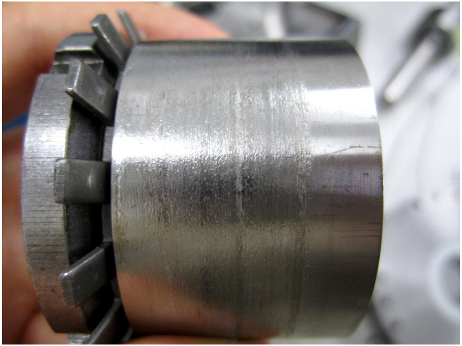
Thank you for your attention

Backward Whirl

Very damaging: Rotor cannot be recovered and hence immediate shutdown necessary.
[Keogh & Cole, 2003]



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