Introduction to integrable quantum field theory

Hilary term 2006, week 3 to 8, Friday 11am, Fisher room, Denis Wilkinson Building

Benjamin Doyon
Rudolf Peierls Centre for Theoretical Physics
Oxford University

Synopsis and Plan

In these lectures, I will decribe the basic concepts of integrable quantum field theory. The general subject of integrability is very old, and many approaches have been developed to study integrable models of many kinds. I will focus on a small, yet important, subset of models and approaches: (2-dimensional) integrable models of quantum field theory (QFT), and the approach that relies on principles of QFT. Models of QFT have the advantage, over classical statistical models on lattices or quantum chains, of describing the universal physics near second-order (classical or quantum) phase transitions. The QFT approach, on the other hand, has the advantage of clarifying the structure of QFT in general and of providing tests for some of its principles. Moreover, QFT is notorious for eluding rigorous mathematical studies, except for free models and conformal field theory (CFT). Models of integrable QFT offer a rare opportunity for more precise and self-consistent descriptions.

1. (1 lecture) I will start by recalling the main roles of general QFT (although I will mainly stay in 1+1 dimensions for simplicity) as a theory for the scaling limit of certain statistical or quantum models and as a theory of relativistic particles. I will describe (or recall) the associated **basic concepts** that provide a "bigger picture" and that are important in integrable QFT:
   - in the "on-shell" description, **the asymptotic states (forming a Hilbert space) and the scattering matrix**;
   - in the "off-shell" description, **the local fields and the operator product expansion (giving the operator algebra)**;
   - the main objects of QFT, **the correlation functions**, which essentially relate both descriptions.
2. (4 lectures) Then I will go to the main properties of integrable QFT as a **factorized scattering theory**:
   - (1 lecture) from the presence of an infinite number of conservation laws: the **factorization of the scattering matrix**, its simple **analytical properties** and the **Yang-Baxter equations**, and how to determine the scattering matrix from these properties, some intuition and some verifications by standard methods;
   - (1 lecture) from the knowledge of the scattering matrix: the **form factor equations** (they form a Riemann-Hilbert problem) for the matrix elements of local fields in the basis of asymptotic states (the form factors);

- (2 lectures) **some methods to solve these equations** and calculate the form factors: this gives in fact a quite explicit representation of the operator algebra on the Hilbert space, and allows to **reconstruct the correlation functions** from the form factors.

I will illustrate these properties with simple models as necessary, I may give an example of application of the correlation functions obtained (for instance, the spectral density in the Hubbard model near to its critical point) and I may describe some of the underlying mathematical structure.

3. (1 lecture, optional) This can already be a lot, but depending on how it goes, I can touch upon other subjects as desired (although each of them could form many other lectures if developed):

   - integrable QFT with boundaries;
   - the non-linear differential equations to determine correlation functions in the Ising model (or: Painlevé equations and solving the associated connection problems from Clifford algebras);
   - Bethe ansatz techniques;
   - perturbing an integrable model;
   - conformal perturbation theory;
   - the correlation functions in integrable QFT at finite temperature...

I will adjust the level of my course to the backgrounds of the participants. For instance, it is OK for me not to assume more than a basic knowledge of QFT (a knowledge of free theories and some idea of what may happen when there is interaction could be sufficient). Also, I can go to more or less technical details and more or less involved mathematical description depending on what is preferred. I will try to point throughout to yet unresolved problems of integrable quantum field theory. I expect to give 5 lectures (possibly extendable to 6).