Non-equilibrium steady states in (near) critical quantum systems

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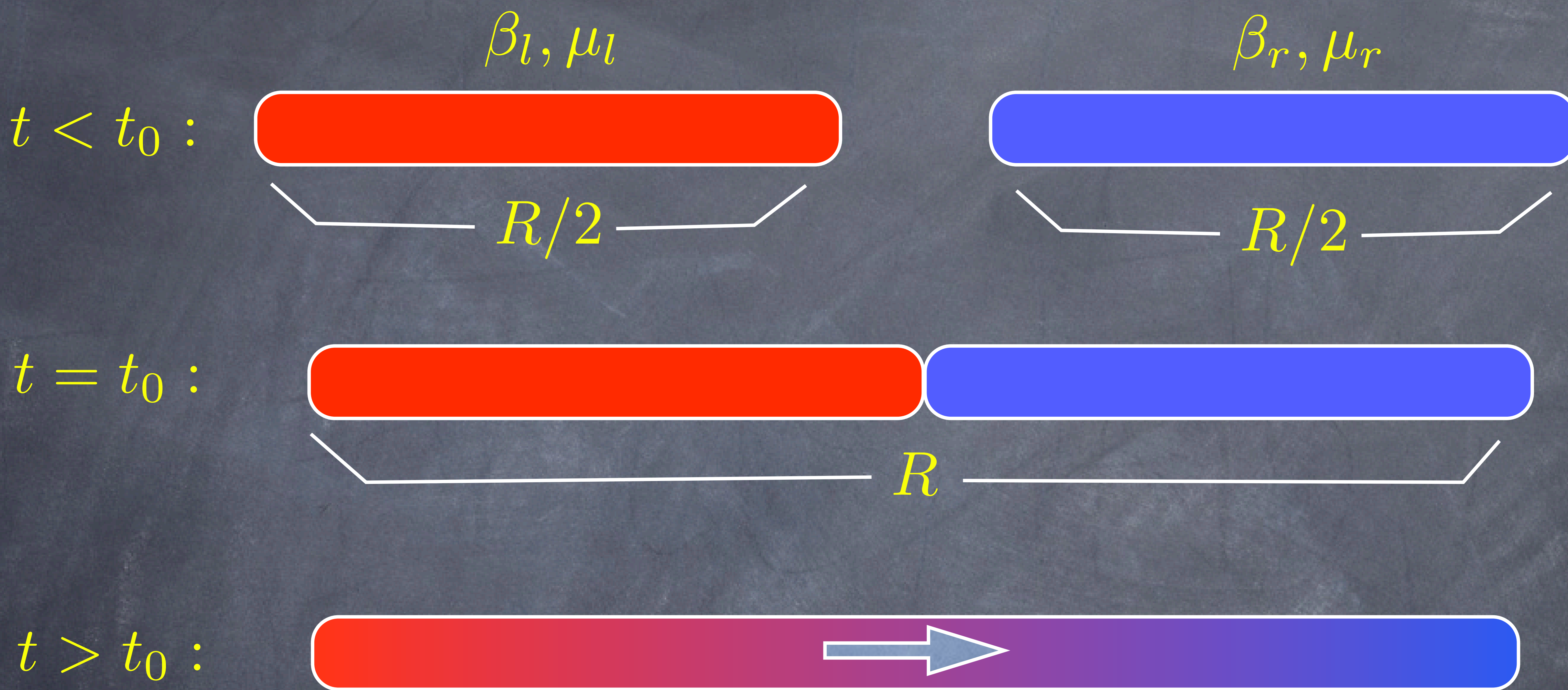
based on works (some in preparation) with
Denis Bernard, Olalla Castro Alvaredo, Yixiong Chen, Andrea De Luci, Fabian Essler, Marianne Hoogeveen, Jacopo Viti

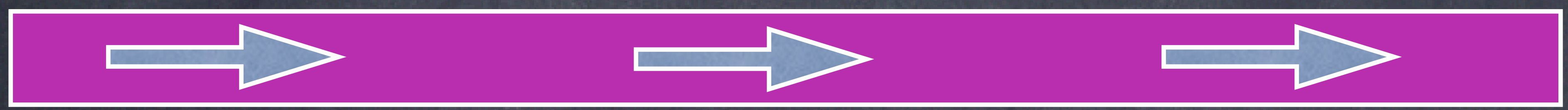
- J. Math. Phys. 53 (2012) 122302, arXiv:1105.1695
- J. Phys. A: Math. Theor. 45 (2012) 362001, arXiv:1202.0239
- arXiv:1212.1077
- arXiv:1302.3125
- arXiv:1305.0518

Birmingham, 9 May 2013

Physical situation



$$R \gg (t - t_0) v_F \gg \text{observation lengths}$$



Physical situation

Initially: $\rho_0 = e^{-\beta_l(H^l+\mu_l N^l)-\beta_r(H^r+\mu_r N^r)}$

Evolution: $H = H^l + H^r + H_{\text{contact}}$

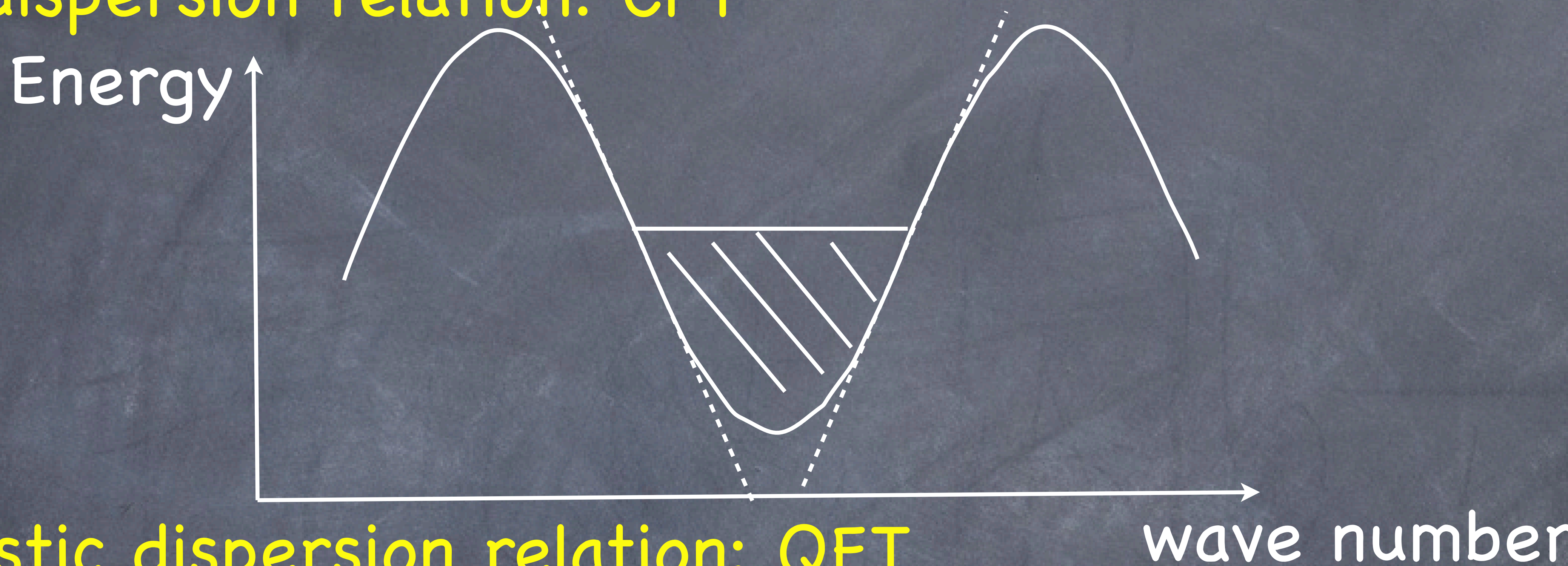
Steady state limit:

$$\langle \cdots \rangle_{\text{ness}} = \lim_{t_0\to-\infty} \lim_{R\to\infty} \frac{\mathrm{Tr}\left(e^{iHt_0}\rho_0 e^{-iHt_0}\ldots\right)}{\mathrm{Tr}\left(\rho_0\right)}$$

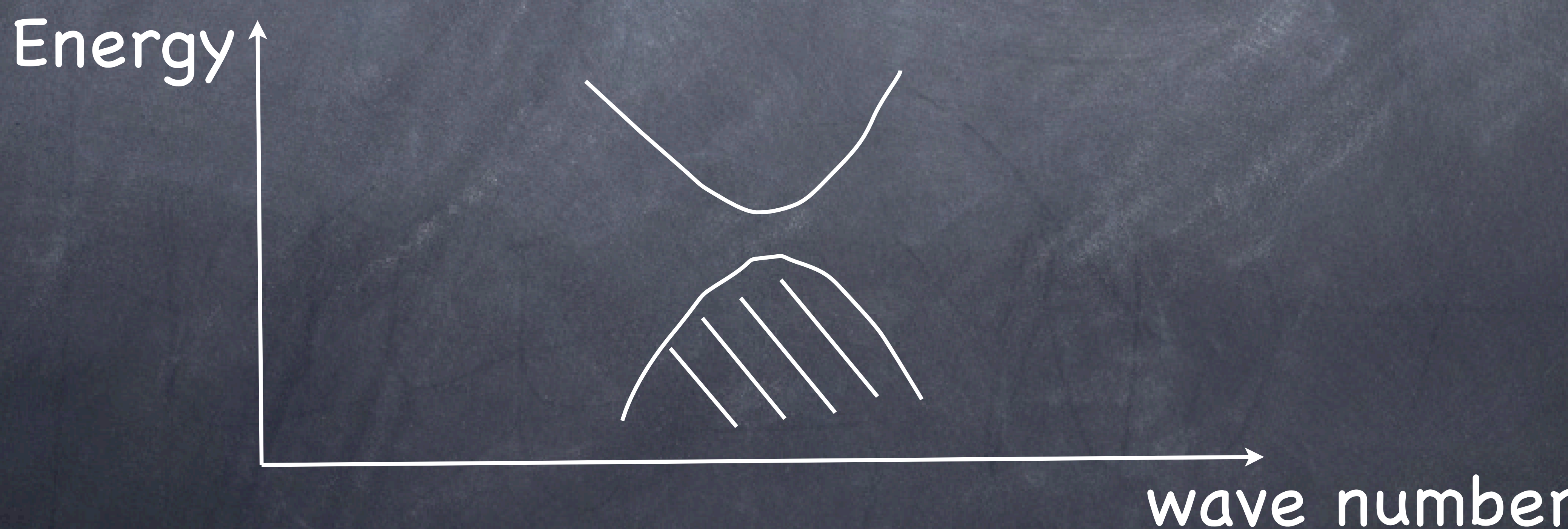
Observables supported on a finite region

Scaling limit: (relativistic) QFT

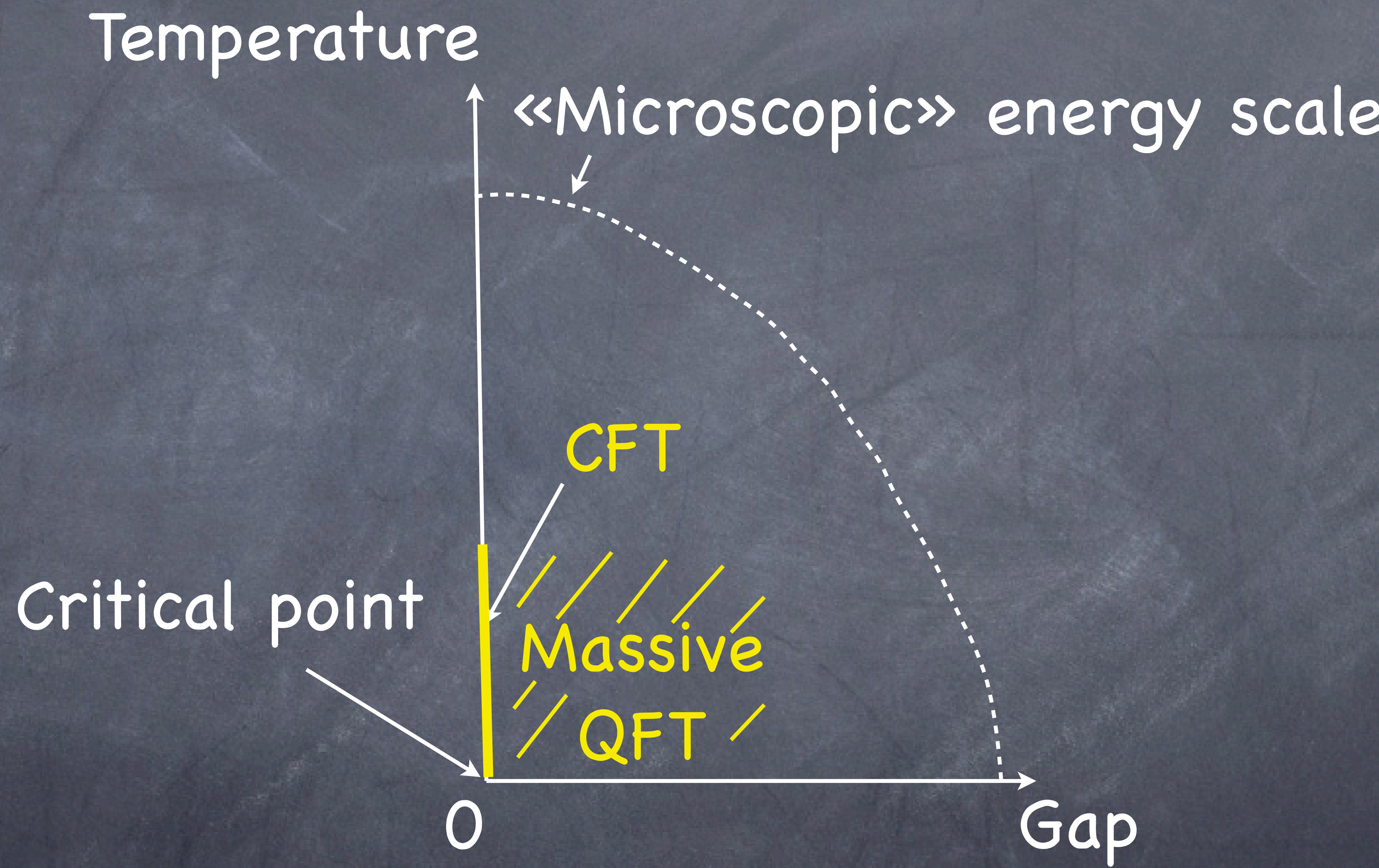
Linear dispersion relation: CFT



Relativistic dispersion relation: QFT



Scaling limit: (relativistic) QFT



Formal description of steady state

in massive QFT
(temperature difference only)

[D. Bernard & BD]
[BD]

$$\boxed{\langle\cdots\rangle_{\rm ness} = {\rm Tr}\left(e^{-W}\cdots\right)\ /\ {\rm Tr}\left(e^{-W}\right)}$$

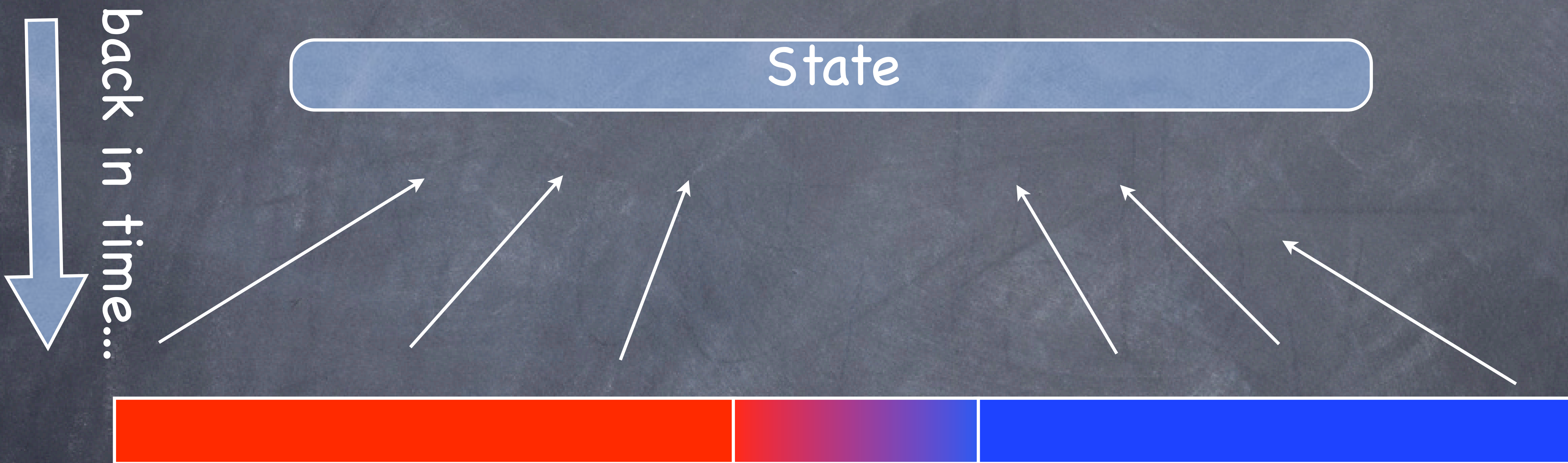
$$W = \beta_l \int_0^\infty d\theta E_\theta n_\theta + \beta_r \int_{-\infty}^0 d\theta E_\theta n_\theta$$

Total energy of right-moving asymptotic particles

Total energy of left-moving asymptotic particles

*In massive QFT: states are described by asymptotically free particles characterized by their rapidities

Formal description of steady state



agreement with free fermion calculations
[W. H. Aschbacher & C.-A. Pillet, 2003]

Formal description of steady state

in CFT

(temperature difference, but more general case known)

[D. Bernard & BD]

$$\boxed{\langle\cdots\rangle_{\rm ness} = {\rm Tr}\left(e^{-W}\cdots\right)\ /\ {\rm Tr}\left(e^{-W}\right)}$$

$$W = \beta_l \frac{L_0}{2\pi R} + \beta_r \frac{\bar L_0}{2\pi R} \qquad (R\to\infty)$$

Total energy of right-movers   Total energy of left-movers

*In CFT: densities separate into right-movers and left-movers

energy density: $h(x) = h_+(x) + h_-(x)$

momentum density: $p(x) = h_+(x) - h_-(x)$

Approach to steady state

in CFT

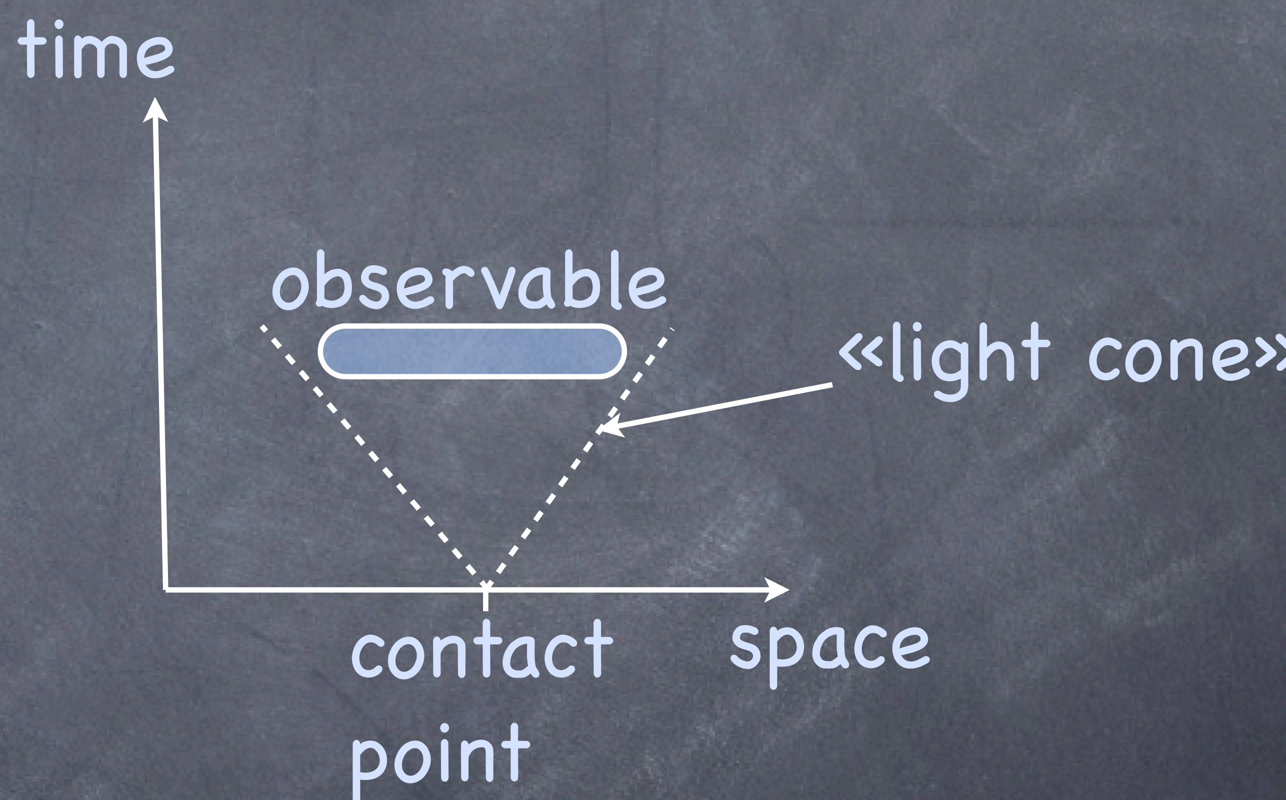
(observables being symmetry currents and descendants)

[D. Bernard & BD]

Two time scales:

- Universal time scale

$$\tau_{\rm uni} = \frac{\text{observation length}}{v_F}$$



- Microscopic time scale (light cone not sharp)

agreement with numerics [C Karrasch, R. Ilan & J. E. Moore, 2012]

The energy current in CFT

Using well known character formulas in order to calculate

$$\langle p(0)\rangle_{\mathrm{ness}} = \frac{\mathrm{Tr}\left(e^{-\beta_l \frac{L_0}{2\pi R}} h_+(0)\right)}{\mathrm{Tr}\left(e^{-\beta_l \frac{L_0}{2\pi R}}\right)} - \frac{\mathrm{Tr}\left(e^{-\beta_r \frac{L_0}{2\pi R}} h_-(0)\right)}{\mathrm{Tr}\left(e^{-\beta_r \frac{\bar{L}_0}{2\pi R}}\right)}$$

in the limit $R \to \infty$

The energy current in CFT

[D. Bernard & BD]

$$\boxed{J_{\Delta\mu=0} = \frac{\pi c}{12}(\beta_l^{-2} - \beta_r^{-2}) = \frac{\pi c k_B^2}{12\hbar}(T_l^2 - T_r^2)}$$

central charge of Virasoro algebra

$$h_+(x) \propto -\frac{c}{24} + \sum_{n\in Z} L_n e^{-\frac{2\pi i n x}{R}}$$

Virasoro

$$\boxed{J_{\Delta\beta=0} = \frac{1}{8\pi\hbar}(\mu_l^2 - \mu_r^2)}$$

The energy current in CFT

Agreement with other works

- confirmed by numerics

[C Karrasch, R. Ilan & J. E. Moore, 2012]

- agreement with Luttinger liquid results

[M. Mintchev & P. Sorba, 2012]
[D. B. Gutman, Yu. Gefen & A. D. Mirlin, 2010]

Fluctuations

We want to measure the fluctuations of transfer, with the transferred quantity taken as, e.g. for energy transfer:

$$Q = \frac{1}{2}\left(H^r - H^l\right)$$

$$Q = q_0$$

$$Q = q_0 + q$$

Fluctuations

$$P(q,t) = \sum_{q_0} \mathrm{Tr}\left(P_{q_0+q} e^{-iHt} P_{q_0} \rho P_{q_0} e^{iHt} P_{q_0+q}\right)$$

Starting point:
either in steady state ρ_{ness}
or at the connection time ρ_0

Fluctuations

$$P(\lambda,t) = \sum_q e^{i\lambda q} P(q,t)$$

$$\boxed{\log P(\lambda,t) \sim t F(\lambda) + O(1)}$$

Cumulant generating function

$$= i\lambda J + \dots$$

Fluctuations in CFT

[D. Bernard & BD]

Independently of starting point for charge transfer
Starting point at connection time for energy transfer
(or: indirect measurement (cf Levitov & Lesovik, 1993))

$$P(\lambda,t) = \frac{\mathrm{Tr}\left(\rho_{\mathrm{ness}}\, e^{i\lambda Q(t)} e^{-i\lambda Q}\right)}{\mathrm{Tr}\,\rho_{\mathrm{ness}}} \qquad (t\to\infty)$$

Fluctuations in CFT

[D. Bernard & BD]
[D. Bernard, BD, M. Hoogeveen]

Measuring fluctuations of both energy and U(1) charge:

λ ν

$$F(\lambda,\nu) = f(\lambda,\nu;\beta_l,\mu_l) + f(-\lambda,-\nu,\beta_r,\mu_r)$$

$$f(\lambda,\nu;\beta,\mu) = \frac{c\pi}{12\hbar}\left(\frac{1}{\beta - i\lambda} - \frac{1}{\beta}\right) + \frac{(\beta\mu + i\nu)^2}{8\pi\hbar(\beta - i\lambda)} - \frac{\beta\mu^2}{8\pi\hbar}$$

Gaussian for charge transfer only
Poissonian for energy transfer only

Fluctuation relation

$$\boxed{F(\lambda,\nu) = F(i(\beta_r-\beta_l)-\lambda, i(\beta_l\mu_l-\beta_r\mu_r)-\nu)}$$

Equivalent to (e.g. for energy transfer):

$$P(q,t\to\infty) = e^{(\beta_l-\beta_r)q}P(-q,t\to\infty)$$

Similar relation was argued for in: Jarzynski, Wojcik (PRL 2004)

See the nice review by: Esposito, Harbola, Mukamel (RMP 2009)

Basic ideas: Gallavoti, ...

Energy current in massive integrable QFT

Using Thermodybamic Bethe ansatz (Al. B. Zamolodchikov, 1990) in order to evaluate

$$J = \mathrm{Tr}\left(e^{-W} p(0)\right)\ /\ \mathrm{Tr}\left(e^{-W}\right)$$

*In massive integrable QFT: scattering matrix factorizes into two-particle processes, and scattering is elastic, set of rapidities is preserved.

Energy current in massive integrable QFT

[O. Castro Alvaredo, Y. Chen, BD & M. Hoogeveen]

[integrable quantum chains: BD and F. Essler]

We find: $J = \frac{d}{da} f_a \Big|_{a=0}$

$$f_a = -\int \frac{d\theta}{2\pi} m \cosh\theta \, \log\left(1 + e^{-\epsilon_a(\theta)}\right)$$

$$\epsilon_a(\theta) = W(\theta) + a\, m \sinh\theta - \int \frac{d\gamma}{2\pi} \varphi(\theta-\gamma) \, \log\left(1 + e^{-\epsilon_a(\gamma)}\right)$$

One-particle eigenvalues of W

$$\varphi(\theta) = -i \frac{d}{d\theta} S(\theta)$$

Two-particle scattering matrix

Fluctuations and PT symmetry

[D. Bernard & BD]

If there is dynamical PT symmetry and «asymptotic» PT invariance:

$$-i\frac{d}{d\lambda}F(\lambda) = J(\beta_l - i\lambda, \beta_r + i\lambda)$$

This holds in CFT, and in massive integrable QFT with energy transfer

Something similar for charge current in CFT, and currents associated to other conserved quantities in integrable QFT

Correlation functions in steady state (Ising model)

[Y. Chen & BD]

We use a «spectral decomposition» in an appropriate space (Liouville space) in order to obtain a convergent expansion, valid at large distances:

$$\langle \sigma(x)\sigma(0)\rangle_{\rm ness} = \int d\theta \cdots \, |\text{form factors}|^2 \, e^{imx\sum \sinh\theta}$$

Two-particle form factors:

$$f(\theta_1,\theta_2) \propto h(\theta_1)h(\theta_2)\tanh\left(\frac{\theta_2-\theta_1}{2}\right)$$

$$h(\theta) \propto \exp\left[\int \frac{d\gamma}{2\pi i}\frac{1}{\sinh(\theta-\gamma)}\log\tanh\frac{W(\gamma)}{2}\right]$$

-> Has a branch cut in rapidity space!

We find an oscillatory behaviour at large distances, e.g. in the disordered phase:

$$\frac{e^{-x\mathcal{E}_{\rm ness}}}{mx}\cos(\nu\log(mx)+B)$$

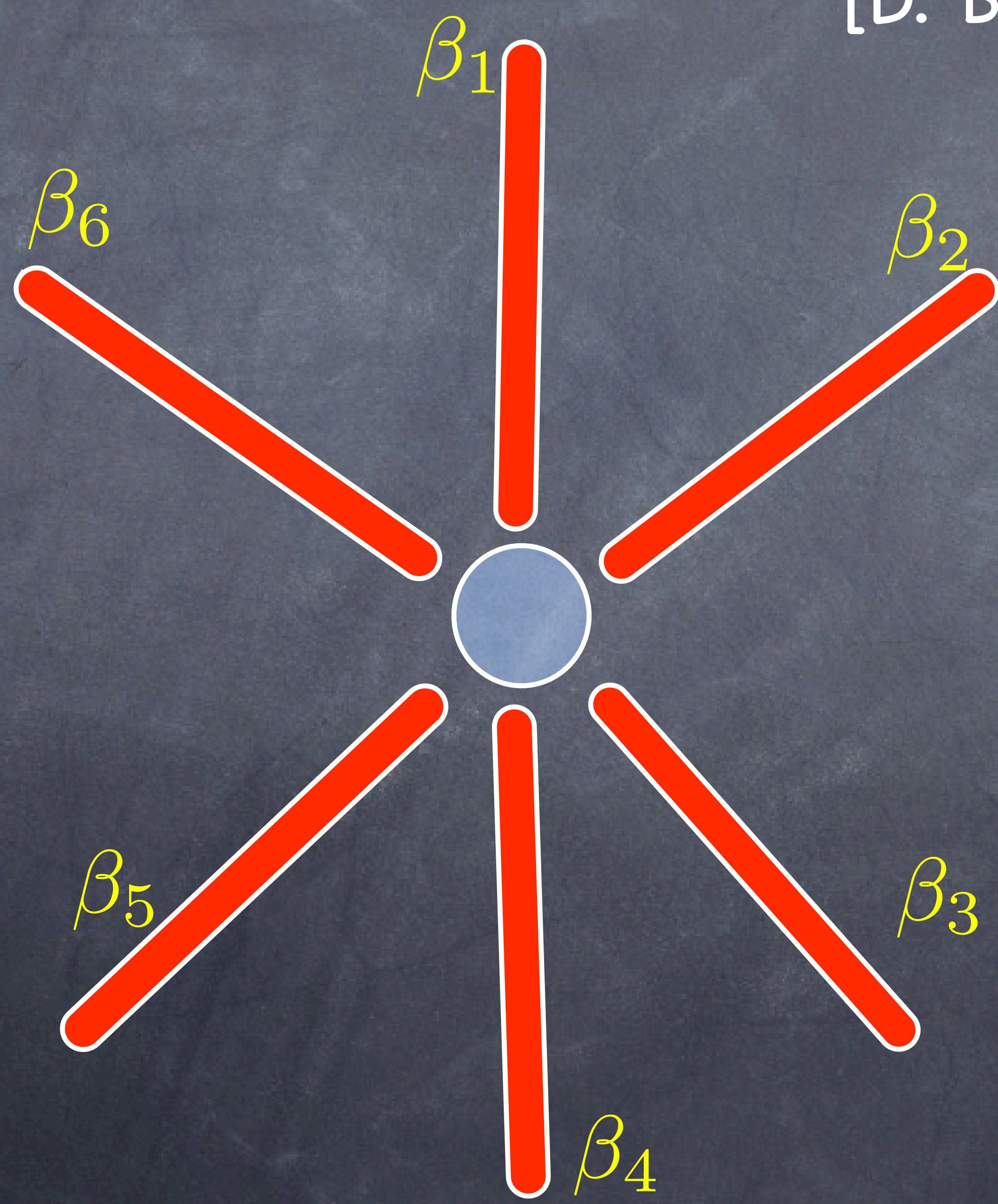
with a frequency determined by the temperatures:

$$\nu=\frac{1}{\pi}\log\left(\coth\frac{m}{2T_r}\ \tanh\frac{m}{2T_l}\right)$$

Star-graphs in CFT...

Generalization of energy current and fluctuations to certain star-graphs in CFT

[D. Bernard, BD & M. Hoogeveen]



Conclusion and perspectives

- Generalization to presence of impurities
- Non-equilibrium correlation functions in integrable models
- Higher dimensions
- Full stochastic interpretation of fluctuation formulas